# Realizing What Might Be[*]

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**Abstract**

Schulz has shown that the suppositional view of indicative conditionals leads to a corresponding view about epistemic modals. But his case backfires: the resulting theory of epistemic modals gets the facts wrong, and so we end up with a good argument against the suppositional view. I show how and why a dynamic view of indicative conditionals leads to a better theory of epistemic modals.

## 1 A Logical Link

Schulz has suggested supplementing the suppositional view of the indicative conditional ($\Rightarrow$) with a corresponding view of the epistemic modals *might* ($\Diamond$) and *must* ($\Box$). This suggestion is motivated by a logical link between epistemic modals and indicative conditionals which has intuitive support and can be proven using some fairly innocent principles:[1]

(L) $\Diamond A \equiv \neg(A \Rightarrow \bot)$, and $\Box A \equiv \neg A \Rightarrow \bot$

(L) ties a theory of epistemic modals to whatever we think is the best theory of indicative conditionals. Fans of the suppositional view of the indicative conditional believe that such conditionals are used to make conditional assertions and are evaluated by conditional probabilities.[2] So given (L), they should also become fans of a corresponding view about epistemic modals. Specifically, (ii) and (iii) are easy to prove on the basis of (i) and (L) under the assumption that logically equivalent statements are assigned the same subjective probability:

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[1]See Schulz (2009), §3 for details. As usual, '$\equiv$' represents logical equivalence and '$\bot$' stands for an arbitrary contradiction.

[2]See, e.g., Adams (1975), Edgington (1995) and Barnett (2006). I follow the standard practice of wedding the suppositional view to conditional probabilities and a standard analysis of these probabilities. In principle, one may hold that conditionals are used to make conditional assertions but insist that they are evaluated by a different method. Such possible views—whatever they are—are *not* the target of my discussion.

(i) $P(A \Rightarrow B) = P(B \mid A)$, where $P(B \mid A) = P(A \wedge B)/P(A)$ if $P(A) > 0$ and $P(B \mid A) = 1$ otherwise[3]
(ii) $P(\Diamond A) = 1$ if $P(A) > 0$ and $P(\Diamond A) = 0$ otherwise
(iii) $P(\Box A) = 1$ if $P(A) = 1$ and $P(\Box A) = 0$ otherwise

This is an important suggestion, but it cuts both ways: if the resulting view of epistemic modals gets the facts wrong, then so much the worse for the suppositional view of conditionals. And that is precisely what we get once we take a closer look at how epistemic modals behave, and in particular at how they interact with conditionals. But fortunately the suppositional view is not the only game in town, and, what is more, there is a competitor—the dynamic view of the indicative conditional—which does justice to (L) *and* gets the logic of epistemic modals right. The purpose of this article is to make good sense of these claims and to explain a bit *why* they make good sense. Very roughly: both the suppositional and the dynamic view implement Ramsey's idea that in order to find out whether 'If $A$, then $B$' is acceptable, one should ask whether $B$ is acceptable under the supposition that $A$.[4] The suppositional view, however, misconstrues how suppositional reasoning works. Specifically, suppositional theorists wrongly predict that rational commitment is always preserved in suppositional reasoning. This prediction causes big trouble when we allow epistemic modals to play with conditionals. The dynamic view, in contrast, gets the basics of suppositional reasoning right and that, as I shall conclude, is why it also gets the facts about epistemic modals right.

## 2 Must Do Better

It is a common observation that epistemically modalized sentences may occur in conditionals:

(1) a. If John is not in Chicago, then he must be in Boston.
b. If we do not invite Bob to the party, he might be disappointed.
c. If it might be raining, I stay at home.

Not only are the examples in (1) well-formed, but they can also figure in valid reasoning. Consider how the first example (1a) performs:

**Case 1** I do not know where John is, but he must be in Chicago or in Boston. *So*, if he is not in Chicago, then he must be in Boston; and if he is not in Boston, then he must be in Chicago.

[3] This principle is just what Edgington (1995) calls 'The Thesis', but extended with the proviso that $P(B \mid A) = 1$ if $P(A) = 0$.
[4] See Ramsey (1990) for details.

**Case 2** If John is not in Chicago, then he must be in Boston. John is not in Chicago. *So*, he must be in Boston.

These are as intuitive entailments as we are likely to find, and every successful story about epistemic modals has to answer to these facts. And of course, the suppositional story has some definitive things to say about these cases, as it offers a unified perspective on both epistemic modals and indicative conditionals. The problem is that what the suppositional view has to say is plain wrong. Let us go through the details.

Suppositional views about conditionals usually rely on Adams's probability logic, and there is no reason to change that good tradition once we have added a probabilistic account of epistemic modals to the picture.[5] Take the *uncertainty* of a sentence as its improbability, which equals 1 minus its probability. The idea is that an argument is probabilistically valid if and only if the uncertainty of the conclusion cannot exceed the sum of the uncertainties of the premises. More precisely, given a language $\mathcal{L}$, we say that $P$ is an *extended probability function* over $\mathcal{L}$ just in case $P$ assigns probabilities to elements of $\mathcal{L}$ in accordance with (i)–(iii) and the following standard laws of probability:

(iv) $0 \leqslant P(A) \leqslant 1$ for all $A$
(v) $P(T) = 1$ for every truth-functional tautology $T$
(vi) $P(A \vee B) = P(A) + P(B)$ if $\neg(A \wedge B)$ is a truth-functional tautology

For each such probability function $P$ we can then derive the corresponding uncertainty function $U_P$ by setting $U_P(A) = 1 - P(A)$ for each $A \in \mathcal{L}$, and then define probabilistic validity as follows:

**Probabilistic Validity** If $\phi_1, \ldots, \phi_n, \psi \in \mathcal{L}$, then $\phi_1, \ldots, \phi_n \therefore \psi$ is probabilistically valid iff there is no extended probability function $P$ over $\mathcal{L}$ such that $U_P(\psi) > \sum_{i=1}^{n} U_P(\phi_i)$.

Schulz shows that this framework gets some basic entailments involving epistemic modals right, but the really interesting question is how the theory performs once we allow epistemic modals and conditionals to play together. And there is not much for us to decide once we are signed up for (i)–(iii) and a standard probability logic: only some minor scope matters have to be sorted out. Consider our sentence (1a) again. When we want to represent 'If John is not in Chicago, then he must be in Boston' in our formal language $\mathcal{L}$, we have to decide what scope the modal takes.[6] The short menu of options include the

[5] See Adams (1975) and Adams (1998).

[6] I set aside the suggestion that 'If John is not in Chicago, then he must be in Boston' is just equivalent to the plain 'If John is not in Chicago, then he is in Boston'. This suggestion is poorly motivated since it is hard to see why *must* should add nothing to what is expressed by an utterance of (1a). Moreover, the argument against the widescoped analysis below also applies when we interpret 'If John is not in Chicago, then he must be in Boston' as a plain conditional. Hence we do not need to consider this suggestion explicitly.

narrowscoped (2a) or widescoped analysis (2b), where $A$ is short for 'John is in Chicago' and $B$ is short for 'John is in Boston':

(2) a. $\neg A \Rightarrow \Box B$
b. $\Box(\neg A \Rightarrow B)$

The claim that the modal *must* takes wide or narrow scope on the level of logical form does *not* presuppose that conditionals express propositions. Instead, the wide- and narrowscoped analyses capture the two ways in which Schulz may interpret (1a).[7] On the narrowscoped reading, an utterance of (1a) is a conditional assertion of an epistemically modalized sentence: to say that John must be in Boston if he is not in Chicago is to conditionally assert that John must be in Boston under the supposition of his not being in Chicago. Accordingly, on this reading one should accept 'If John is not in Chicago, then he must be in Boston' just in case the conditional probability $P(\Box B \mid \neg A)$ is sufficiently high.

On the widescoped reading, an utterance of (1a) expresses certainty in a conditional: to say that John must be in Boston if he is not in Chicago is to say that, certainly, John is in Boston under the supposition of his not being in Chicago. In other words, an utterance of (1a) expresses conditional probability 1 for John being in Boston given that he is not in Chicago. Accordingly, on this reading one should accept 'If John is not in Chicago, then he must be in Boston' just in case the conditional probability $P(B \mid \neg A)$ is 1.[8]

Both the narrowscoped and the widescoped analysis of (1a) are intelligible attempts at capturing what we express when we utter 'If John is not in Chicago, then he must be in Boston'. But whatever option we choose, the suppositional view will get at least one of the basic facts wrong. Suppose that narrowscoping is the way to go. Then what is going on in the first case looks as follows in $\mathcal{L}$:

(3) a. John must be in Chicago or in Boston.
$\Box(A \vee B)$
b. If John is not in Chicago, then he must be in Boston.
$\neg A \Rightarrow \Box B$
c. If John is not in Boston, then he must be in Chicago.
$\neg B \Rightarrow \Box A$

[7]Though Schulz does not explicitly consider examples such as (1a), the suggestions are in the spirit as well as the letter of his proposal. Schulz effectively admits that modals may scope over conditionals in fn. 9 of his article, and has no reason to deny that modals may scope over consequents.

[8]What has been said about (2a) and (2b) applies to the the narrowscoped and widescoped reading of a sentence like 'If John is not in Chicago, he might be in Boston' as well. On the narrowscoped analysis, to utter that sentence is to conditionally assert that John might be in Boston under the supposition of his not being in Chicago. On this reading one should accept 'If John is not in Chicago, then he might be in Boston' just in case $P(\Diamond B \mid \neg A)$ is sufficiently high. On the widescoped analysis, an utterance of 'If John is not in Chicago, he might be in Boston' expresses non-zero confidence in 'If John is not in Chicago, then he is in Boston'. On this reading one should accept that sentence just in case $P(B \mid \neg A)$ is greater than zero.

If narrowscoping is right, then every good story about epistemic modals has to predict that the step from (3a) to (3b) and (3c) is valid. The suppositional view falls short of this requirement (we can safely ignore (3c) from now on):

**Fact 1** $\Box(A \vee B) \therefore \neg A \Rightarrow \Box B$ is not probabilistically valid.

*Proof* Suppose $P(A) = 0.5 = P(B)$, $P(A \vee B) = 1$. Then $P(\Box(A \vee B)) = 1$ but $P(\Box B) = 0$ and so $P(\Box B \wedge \neg A)/P(\neg A) = 0$. Accordingly, $U(\neg A \Rightarrow \Box B) > U(\Box(A \vee B))$, and thus $\Box(A \vee B) \therefore \neg A \Rightarrow \Box B$ is not probabilistically valid. $\Box$

The problem with the narrowscoped reading is as follows. One may be certain that John is in Chicago or in Boston and yet be uncertain of his being in Boston and uncertain of his being in Chicago. In that case the probability of (3a) is 1 and the probability of (3b) on its narrowscoped reading is 0, which is just to say that the step from the former to the latter is probabilistically invalid. This is already very bad news for the suppositional view, since narrowscoping just *seems* to be the way to go. But even choosing the widescoping option does not help, as it is now the second case that causes trouble. If we opt for widescoping, then the second case looks as follows in $\mathcal{L}$:

(4) a. If John is not in Chicago, then he must be in Boston.
$\Box(\neg A \Rightarrow B)$
b. John is not in Chicago.
$\neg A$
c. John must be in Boston.
$\Box B$

If widescoping is the way to go, then every good theory of epistemic modals has to predict that (4a) and (4b) jointly entail (4c). But again the suppositional view does not get this right:

**Fact 2** $\Box(\neg A \Rightarrow B), \neg A \therefore \Box B$ is not probabilistically valid.

*Proof* Suppose $P(A) = 0.5 = P(B)$, $P(\neg A \wedge B) = 0.5$. Then $P(\neg A \wedge B)/P(\neg A) = 1$ and so $P(\neg A \Rightarrow B) = 1 = P(\Box(\neg A \Rightarrow B))$. But $P(\Box B) = 0$ and so $U(\Box B) > U(\Box(\neg A \Rightarrow B)) + U(\neg A)$, from which the probabilistic invalidity of $\Box(\neg A \Rightarrow B), \neg A \therefore \Box B$ follows. $\Box$

The problem with the widescoped analysis is as follows. One may be certain that John is in Boston if he is not in Chicago and yet be uncertain of his being is in Boston—all it takes is that there is at least some chance that he is in Chicago. Hence on the widescoped reading of (4a) the uncertainty of (4c) may outstrip the sum of the uncertainties of (4a) and (4b). In our example, the combined uncertainty of (4a) and (4b) is 0.5, while the uncertainty of (4c) is 1. Hence the argument is probabilistically invalid if one opts for widescoping.

What makes a principle like (L) interesting is that it ties a theory of epistemic modals to a theory of indicative conditionals. One promising strategy is now to move from a given analysis of indicative conditionals to a corresponding analysis of epistemic modals. But as we have seen this strategy may backfire: combining (L) with the suppositional view of indicative conditionals gets the facts wrong when we look at how conditionals interact with epistemic modals. And assuming that (L) is in good shape, that just means that the suppositional view about indicative conditionals gets the boot as well. Ironically, what was intended as an argument *for* a certain theory of epistemic modals becomes an argument *against* a certain theory of conditionals.

# 3 Might Do Better

The previous considerations would carry much less weight if no one in town could establish a compelling link between epistemic modals and indicative conditionals without shipwreck. But there is such a view. Consider a *dynamic* perspective on meaning and communication which treats the meaning of a sentence as its *Context Change Potential* (CCP). Semantic values then become relational: they relate an input context (i.e. the one in which the sentence is uttered) to an output context (i.e. what the context looks like after the assertion).[9] And think of a context in the way Stalnaker (1978) does: as a set of possible worlds. The CCP or update effect of an atomic sentence is then simply to eliminate all worlds in the context at which the sentence is false, and we can then define the CCP of more complex sentences on that basis. So if $W$ is our set of worlds and $\sigma$ is a context, then we can define our *update function* $[.]\colon \wp(W) \mapsto \wp(W)$ for a simple propositional language as follows:

(vii) $\sigma[p] = \{w \in \sigma \colon w(p) = 1\}$
(viii) $\sigma[\neg\phi] = \sigma \backslash \sigma[\phi]$
(ix) $\sigma[\phi \wedge \psi] = \sigma[\phi][\psi]$

In words, updating with $p$ simply eliminates all worlds at which $p$ is false. Updating with $\ulcorner\neg\phi\urcorner$ is just taking the complement of the result of updating with $\phi$. And updating with $\ulcorner\phi \wedge \psi\urcorner$ is just updating with the first conjunct $\phi$ and then updating with the second conjunct $\psi$.

To see how a conditional might work, remember Ramsey's idea that in order to evaluate a conditional, one should check whether one (hypothetically) accepts the consequent under the assumption that the antecedent holds. So conditionals may be understood as *tests* on the context: to update a context $\sigma$ with $\ulcorner\phi \Rightarrow \psi\urcorner$ is to check whether updating $\sigma$ with $\phi$ and then $\psi$ amounts to nothing more than updating $\sigma$ with $\phi$. That is, it is to test whether the result of updating $\sigma$ with $\phi$ will already contain the information encoded in $\psi$. If this is the case, $\sigma$

[9]Some popular dynamic semantics: Discourse Representation Theory (Kamp (1981); Kamp and Reyle (1993); Kamp et al. (2009)), File Change Semantics (Heim (1982)), Update Semantics (Veltman (1985, 1996)) Dynamic Predicate Logic (Groenendijk and Stokhof (1991)).

passes the test and the update just delivers the original state; if this is not the case, the test has failed and we just get back the empty set.[10]

(x) $\sigma[\phi \Rightarrow \psi] = \{w \in \sigma \colon \sigma[\phi][\psi] = \sigma[\phi]\}$

With such a theory of the indicative conditional in place, we may ask what follows for our theory of epistemic modality, given that the logical equivalences in (L) are just right on the money. Assume that logical equivalence comes down to identity of update effects: two sentences $\phi$ and $\psi$ are logically equivalent just in case updating any context with $\phi$ has the same outcome as updating that context with $\psi$. Then (L) and (x) jointly entail the following update rules for *might* and *must*:

(xi) $\sigma[\Diamond\phi] = \{w \in \sigma \colon \sigma[\phi] \neq \varnothing\}$
(xii) $\sigma[\Box\phi] = \{w \in \sigma \colon \sigma[\phi] = \sigma\}$

*Proof* Observe that our update rules guarantee that $\forall\sigma \colon \sigma[\bot] = \varnothing$ and that $\forall\sigma\forall\phi \colon \sigma[\phi] \subseteq \sigma$. Thus:

$$
\begin{array}{rcll}
\sigma[\Diamond\phi] & = & \sigma[\neg(\phi \Rightarrow \bot)] & \text{(L)} \\
& = & \sigma \backslash \sigma[\phi \Rightarrow \bot] & \text{(viii)} \\
& = & \sigma \backslash \{w \in \sigma \colon \sigma[\phi][\bot] = \sigma[\phi]\} & \text{(x)} \\
& = & \sigma \backslash \{w \in \sigma \colon \sigma[\phi] = \varnothing\} & \text{(tautology)} \\
& = & \{w \in \sigma \colon \sigma[\phi] \neq \varnothing\} & \text{(tautology)}
\end{array}
$$

$$
\begin{array}{rcll}
\sigma[\Box\phi] & = & \sigma[\neg\phi \Rightarrow \bot] & \text{(L)} \\
& = & \{w \in \sigma \colon \sigma[\neg\phi][\bot] = \sigma[\neg\phi]\} & \text{(x)} \\
& = & \{w \in \sigma \colon \sigma[\neg\phi] = \varnothing\} & \text{(tautology)} \\
& = & \{w \in \sigma \colon \sigma \backslash \sigma[\phi] = \varnothing\} & \text{(viii)} \\
& = & \{w \in \sigma \colon \sigma[\phi] = \sigma\} & \text{(tautology)}
\end{array}
$$

□

So, very much as before we can derive a substantive theory of epistemic modals from a substantive conception of indicative conditionals. Like a conditional, *might* and *must* encode a test on the context. The test associated with ⌜$\Diamond\phi$⌝ is passed just in case updating with the prejacent does not result in the empty state—intuitively, just in case the context allows updating with the prejacent. And the test run by ⌜$\Box\phi$⌝ is passed just in case updating with the prejacent does not induce any change in the context at all—intuitively, just in case the context already contains the information encoded by the prejacent.[11]

[10] To appreciate the full glory of a dynamic semantics for indicative conditionals, I recommend that you take a look at the letters of support from Gillies (2004, 2009a,b) and Willer (2009).

[11] This is just how Veltman (1985, 1996) treats epistemic modals, and dynamic theories of indicative conditionals owe a lot of inspiration to his work. But here the direction of inspiration is reversed: starting with a dynamic theory of indicative conditionals and a logical link between such expressions and epistemic modals, we have arrived at Veltman's theory.

The last point on the to-do list is then to determine what it means for an argument to be valid, and here the simple idea is to put a dynamic understanding of this notion to work. A argument is *dynamically valid* just in case updating any context with each premise and then with the conclusion amounts to nothing more than updating that context with each premise—intuitively, just in case updating any context with each of the premises results in a context that already contains the information encoded in the conclusion. More precisely:

**Dynamic Validity** If $\phi_1, \ldots, \phi_n, \psi \in \mathcal{L}$, then $\phi_1, \ldots, \phi_n \therefore \psi$ is dynamically valid iff every context $\sigma$ is such that $\sigma[\phi_1] \ldots [\phi_n][\psi] = \sigma[\phi_1] \ldots [\phi_n]$.

So there is in fact another game in town that does justice to the intuitive link between epistemic modals and indicative conditionals. That is good news, as it shows us how we can live happily without the suppositional view. But the even better news is that our mini-framework takes the hurdles that the suppositional view knocks over. All entailments turn out as desired if we opt for the natural narrowscope reading:

**Fact 3** $\Box(A \vee B) \therefore \neg A \Rightarrow \Box B$ is dynamically valid.

*Proof* Let $\sigma$ be arbitrary, and consider $\sigma[\Box(A \vee B)]$. The desired result follows trivially if $\sigma[\Box(A \vee B)] = \varnothing$, so what remains to be shown is that if $\sigma[\Box(A \vee B)] = \sigma$, then $\sigma[\neg A \Rightarrow \Box B] = \sigma$. By assumption $\sigma[\Box(A \vee B)] = \sigma$, and so $\sigma[A \vee B] = \sigma$. By the standard translation of '$\vee$', $\sigma[\neg(\neg A \wedge \neg B)] = \sigma$. It follows that $\sigma \backslash \sigma[\neg A \wedge \neg B] = \sigma$ and thus $\sigma[\neg A \wedge \neg B] = \varnothing$. Whence $\sigma[\neg A][\neg B] = \varnothing$, from which it follows that $\sigma[\neg A] \backslash \sigma[\neg A][B] = \varnothing$. Thus $\sigma[\neg A][B] = \sigma[\neg A]$ and hence $\sigma[\neg A][\Box B] = \sigma[\neg A]$. It follows that $\sigma[\neg A \Rightarrow \Box B] = \sigma$, as required. Since $\sigma$ was arbitrary, this completes the proof.[12] $\Box$

**Fact 4** $\neg A \Rightarrow \Box B, \neg A, \therefore \Box B$ is dynamically valid.

*Proof* Let $\sigma$ be arbitrary and assume that $\sigma(\neg A \Rightarrow \Box B) = \sigma$ (again the proof is trivial is the test is not passed). Then $\sigma[\neg A][\Box B] = \sigma[\neg A]$. And that is already what is needed. $\Box$

# 4 Diagnosis and Outlook

Supplementing the suppositional view of indicative conditionals with a corresponding view of epistemic modals yields wrong results. Supplementing the

[12]Things are even better than that, as it is also easy to prove that $A \vee B \therefore \neg A \Rightarrow \Box B$ is dynamically valid. And that makes perfect sense, as the following has the ring of intuitive entailment as well: I don't know where John is, but he is either in Chicago or in Boston. *So* if he is not in Chicago, then he must be in Boston; and if he is not in Boston, then he must be in Chicago.

dynamic view of indicative conditionals with a corresponding view of epistemic modals gets the facts right. This is more than a sheer coincidence, and it is important to see why. For the suppositional view, evaluating a conditional $\ulcorner\phi \Rightarrow \psi\urcorner$ proceeds via *conditionalization*: given a probability distribution $P$, form a derived probability function $P_\phi$ by setting, for each $\chi \in \mathcal{L}$, $P_\phi(\chi) = P(\chi)/P(\phi)$ if $P(\phi) > 0$, and $P_\phi(\chi) = 1$ if $P(\phi) = 0$. The derived probability function $P_\phi$ will just determine the probability of $\ulcorner\phi \wedge \psi\urcorner$ under the supposition that $\phi$. But conditionalization is *conservative*: whenever $P(\chi) = 1$, then $P_\phi(\chi) = 1$.[13] And that explains why the suppositional view does not support the desired inference from '$\Box(A \vee B)$' to '$\neg A \Rightarrow \Box B$'. The issue is that John *might* not be in Boston, and thus $P(\Diamond\neg B) = 1$ and $P(\Box B) = 0$. Since conditionalization is conservative, $P_{\neg A}(\Diamond\neg B) = 1$ and thus $P_{\neg A}(\Box B) = 0$. In other words, the supposition that John is not in Chicago preserves my low credence in '$\Box B$'—hence my low credence in '$\neg A \Rightarrow \Box B$'.

The dynamic perspective, in contrast, gets things right since updating fails to be conservative in the right way. Let us say that $\sigma \vDash \Diamond\phi$ just in case $\sigma[\Diamond\phi] = \sigma$. Updating with $\psi$ is not guaranteed to be conservative as there are $\sigma$ such that $\sigma \vDash \Diamond\phi$ but $\sigma[\psi] \nvDash \Diamond\phi$—just consider $\sigma = \{w_1, w_2\}$ such that $w_1(p) = 1$ and $w_2(p) = 0$, and let $\phi \doteq p$ and $\psi \doteq \neg p$. So established epistemic possibilities are not guaranteed to be preserved when we suppose the antecedent of a conditional, and it is this feature that gets the story straight. Granted, John might be in Chicago, and so he does not have to be in Boston. But once we have updated with 'John is not in Chicago' to accommodate the antecedent of our conditional, we land in a derived context in which this possibility is out, and relative to this derived context, John must be in Boston.

So there is something fundamentally right with the dynamic view and something fundamentally wrong with the suppositional view. Both views correctly assume that in order to evaluate a conditional, we ask whether $B$ is acceptable under the supposition that $A$. But supposing that $A$ is the case does not work the way that suppositional theorists think it does, and the case of epistemic modals shows where exactly they go wrong. On the other hand, a dynamic theory gets the basics of suppositional reasoning right. All of this suggests that when we hope to exploit the intuitive equivalences in (L) to develop a successful theory of epistemic modals, it is the dynamic view of indicative conditionals that should serve as our starting point.

The basic dynamic framework requires some extensions to yield a fully successful theory of conditionals and epistemic modals, including a semantics for tense and a pragmatic theory. This is not the place to dive into the details, but let me quickly address one prominent issue. An anonymous referee remarks that the basic dynamic proposal does not provide for the potential uncertainty of conditionals. For all $\sigma$, either $\sigma \vDash (\phi \Rightarrow \psi)$ or $\sigma \vDash \neg(\phi \Rightarrow \psi)$, which is just to say that a conditional is either fully rejected or fully accepted in a given state (and similarly for epistemic modals). The referee's observation is a fair one,

[13] See Gärdenfors (1982), whose formulation slightly differs since he assumes that conditionalization is undefined if $P(\phi) = 0$.

but a minimal extension of what has been said so far can take it into account. Suppose that instead of changing Stalnakerian contexts, sentences of $\mathcal{L}$ change *states*, which are defined as sets of sets of possible worlds. If $\Sigma$ is such a state, then a new update function $\uparrow : \wp(\wp(W)) \mapsto \wp(\wp(W))$ for $\mathcal{L}$ is defined as follows:

(xiii) $\Sigma \uparrow \phi = \{\sigma : \sigma \neq \varnothing \wedge \exists \sigma' \in \Sigma : \sigma'[\phi] = \sigma\}$

Update of a state $\Sigma$ with a formula $\phi$ thus comes down to the following procedure: first update every element of $\Sigma$ with $\phi$; then gather all the resulting sets of possible worlds, leaving out those that are empty. This determines the output state. Updating an element of $\Sigma$ just proceeds as defined in the previous section, and the derivation of (xi) and (xii) on the basis of (vii)-(x) and (L) proceeds as before.

In the light of (xiii), we can now redefine what it means for an argument to by dynamically valid. An argument has this property just in case updating any *state* with each premise and then with the conclusion amounts to nothing more than updating that state with each premise:

**Dynamic Validity (revised version)** If $\phi_1, \ldots, \phi_n, \psi \in \mathcal{L}$, then $\phi_1, \ldots, \phi_n \therefore \psi$ is dynamically valid iff every state $\Sigma$ is such that $\Sigma \uparrow \phi_1 \ldots \uparrow \phi_n \uparrow \psi = \Sigma \uparrow \phi_1 \ldots \uparrow \phi_n$.

We now say that $\Sigma \models \phi$ just in case $\Sigma \uparrow \phi = \Sigma$. It is trivial to verify that the extended framework preserves the intuitive entailments that I discussed in the previous sections: **Fact 3** and **Fact 4** also hold on the revised version of dynamic validity. In addition to that, we can now account for the potential uncertainty of conditionals, as there are states $\Sigma$ such that $\Sigma \not\models (\phi \Rightarrow \psi)$ and $\Sigma \not\models \neg(\phi \Rightarrow \psi)$: just think of a state in which some but not all elements pass the test encoded by $\ulcorner \phi \Rightarrow \psi \urcorner$. By the same reasoning, we can also provide for the potential uncertainty of epistemic modals, which is illustrated by an example from DeRose (1991). A cancer test is run, and depending on the outcome the patient may or may not have cancer. If the test results are unknown it may be uncertain whether

(5) It might be that N.N. has cancer.

Schulz admits that in order to account for DeRose's example, he needs to postulate an ambiguity: a *might*-statement may either express subjective or objective probability. No such ambiguity is needed in the extended dynamic framework, as there are states $\Sigma$ such that $\Sigma \not\models \Diamond\phi$ and $\Sigma \not\models \neg\Diamond\phi$: just think of a state in which some but not all elements pass the test encoded by $\ulcorner \Diamond\phi \urcorner$. This just illustrates another advantage of the dynamic over the suppositional account of conditionals and epistemic modals.

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