Negating Conditionals*

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Abstract

A recurring narrative in the literature on conditionals is that the empirical facts about negated *ifs* provide compelling evidence for the principle of Conditional Excluded Middle and sit uncomfortably with a large family of analyses of conditionals as universal quantifiers over possible worlds. I show that both parts of the narrative are in need of a rewrite. I do so by articulating an innovative conditional analysis in a bilateral semantic setting that takes inspiration from the Ramsey test for conditionals but distinguishes the classical Ramseyan question of what it takes to *accept* a conditional from the one of what it takes to *reject* a conditional. The resulting framework disentangles the empirical facts about negated conditionals from the validity of Conditional Excluded Middle but also shows how the principle can live happily in a strict analysis of conditionals, and in fact how it can co-exist with other non-classical principles such as Simplification of Disjunctive Antecedents without negative side effects.

1 The Plot

A recurring narrative in the literature on conditionals is that there is an unhappy choice to be made between a plausible semantic analysis of what conditionals say and an empirically adequate story of how conditionals play with negation. The dilemma, in brief, is that we have excellent reasons to think that conditionals are at their semantic core universal quantifiers over a contextually determined set of possible worlds—they require, very roughly, that all relevant antecedent-verifying worlds are also consequent-verifying worlds—and yet such an analysis fails to make sense of why ordinary speakers have the habit of interpreting negated conditionals as conditional negations. My goal here is to demonstrate that this familiar narrative is in need of a rewrite.

The trouble with negated conditionals is a central theme in the ongoing debate about what to make of the principle of Conditional Excluded Middle (CEM):

$$\text{(CEM)} \qquad \vDash (\phi > \psi) \vee (\phi > \neg\psi)$$

It is a familiar fact that—without non-classical maneuvers anyway—CEM fails to be valid in a universalist setting, since whenever some of the relevant ϕ-worlds make ψ true while others make ψ false, neither $\ulcorner \phi > \psi \urcorner$ nor $\ulcorner \phi > \neg\psi \urcorner$ will turn out to be true. The no less

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familiar arguments to the conclusion that this is a problem rather than a boon reliably appeal to the empirical fact that natural language *ifs* fail to enter into scope relations that seemingly are available if CEM is rejected—the interaction between conditionals and negation being the key case in point.

The basic observation that matters here is that conditionals such as (1a) and (1b) ring equivalent:

(1) a. It's not the case that if John takes the exam, he will pass.
b. If John takes the exam, he won't pass.

The challenge is to explain why (1a) should entail (1b): given CEM, this is just an instance of disjunctive syllogism. In contrast, the inference is a bit of a puzzler if the truth of a conditional requires all relevant antecedent-verifying worlds to be consequent-verifying worlds, since the existence of a relevant world at which John takes the exam and fails is perfectly consistent with the existence of a relevant world at which John takes the exam and passes.[1]

That negation causes trouble in a universalist setting is already noted by Lewis (1973), who ends up rejecting CEM but also bemoans that doing so precludes fully accounting for how conditionals play with negation in natural language. Starting with Stalnaker (1981), CEM enthusiasts have repeatedly pressed the latter point in the literature on conditionals, with a variety of glosses.[2] One recent version of the story puts items that lexicalize negation into the spotlight (see Cariani and Goldstein forthcoming and Santorio 2017). Consider the following pair:

(2) a. I doubt that if John takes the exam, he will pass.
b. I believe that if John takes the exam, he won't pass.

(2a) and (2b) have the air of equivalence. That makes good sense if CEM is true, for then rejecting the claim that John will pass if he takes the exam immediately amounts to accepting the claim that he will not pass if he takes the exam. But it is not at all obvious how the equivalence can be explained if CEM is rejected.

Another plea for CEM highlights the interaction between quantifiers and conditionals (see von Fintel and Iatridou 2002, Higginbotham 2003, Klinedinst 2011, and Williams 2010). Observe that (3a) and (3b) sound equivalent:

(3) a. No student will succeed if he goofs off.
b. Every student will fail if he goofs off.

Assume CEM and that the quantifiers scope over the conditionals (and that not succeeding is the same as failing). Take any student and assume that it is not so that he will succeed

[1]The direction from (1b) to (1a) is unproblematic if we assume that conditionals require that their domain of quantification include at least one antecedent-verifying world: if John fails at every antecedent verifying-world he obviously cannot pass at every such world, unless the quantification is vacuous. The assumption at play here has been articulated for conditionals of all stripes (see, e.g., Stalnaker 1975, von Fintel 2001, and Gillies 2007).

[2]Thomason (2012), reflecting on the interaction between conditionals and negation in natural language, characterizes the linguistic evidence for CEM "overwhelmingly strong." It is this kind of evidence that I am interested in right now, though we will consider at least one other type of argument at a later stage (see also, e.g., Cross 2009 for relevant discussion).

if he goofs off: then by CEM he will not succeed and thus fail if he goofs off, and it is thus no wonder that (3a) and (3b) seem equivalent.

Finally, the interaction between *if* and *only* is sometimes taken to provide evidence for CEM:

(4) a. The flag flies only if the Queen is home.
b. If the flag flies, then the Queen is home.
c. The flag flies if the Queen isn't home.

(4a) entails (4b), and this can be explained compositionally if CEM holds, as von Fintel (1997) details (see also Barker 1993). Here the assumption is that *only* in (4a) takes wide scope over the conditional, thus negating the alternatives to the conditional "the flag flies if the Queen is home," which are assumed to include (4c). Together with Contraposition, CEM and the negation of (4c) together imply (4b).

The first point of the present exercise is to demonstrate that the previously sketched narrative about conditionals, negation, and excluded middles is prone to disruption: negated conditionals are read as conditional negations alright, but we can explain all this in a universalist setting without joining the CEM fan club. The second point is that the apparent conflict between embracing CEM and treating conditionals as universal quantifiers over a contextually determined set of possible worlds is unreal: Conditional Excluded Middle can live happily in a universalist setting, and it can in fact co-exist with other non-classical principles such as Simplification of Disjunctive Antecedents without causing unwelcome side effects. The classical narrative on negated conditionals and conditional negations, in brief, is prone to disruption.

My plan is as follows. I begin by describing a theory of conditionals that makes good empirical sense of negated conditionals without endorsing the principle of Conditional Excluded Middle (Section 2). The semantics is heavily inspired by the Ramsey test for conditionals but also makes a case for distinguishing the classical Ramseyan question of what it takes to *accept* a conditional from the one of what it takes to *reject* a conditional. Once we have articulated this distinction in a formal semantics for *ifs* and *nots* we can take the next step and articulate a notion of logical consequence that validates Conditional Excluded Middle and has a range of other interesting consequences (Section 3). Section 4 comments on a slight restatement of the case for CEM and considers the issue of antecedent strengthening. Section 5 concludes the discussion.

2 Negation for the Ramsey Test

I said that we can make sense of negated conditionals in a universalist setting without embracing CEM, so let me explain why this should be so. One way of bringing oneself into a universalist frame of mind is to go back to basics and look at Ramsey's (1931) proposal for how to evaluate conditionals. Ramsey famously suggested that a conditional is accepted, given some state of information s, just in case its consequent is (hypothetically) accepted in the derived state of information got by strengthening s with the assumption of its antecedent. Putting things a bit more precisely:

$$s \Vdash \phi > \psi \text{ just in case } s[\phi] \Vdash \psi$$

Here "$\Vdash$" denotes the relation of acceptance and $s[\phi]$ is the result of strengthening (or updating) s with the information carried by ϕ. It is uncontroversial that Ramsey's proposal carries more than a grain of truth, and it obviously glosses *if* as a universal quantifier under the reasonable assumption that acceptance amounts to entailment by an information carrier understood as a set of possible worlds. I am interested in the question of how this proposal can be turned into a semantics for conditionals that also has something useful to say about negation. I begin with a discussion of the key ideas (Section 2.1) and then work out the framework in more detail (Section 2.2).

2.1 Basics

The key ideas of my story can be elaborated in a variety of ways but for present purposes I will opt for a *dynamic* implementation, not least because dynamic semantics has a fine tradition of directly translating Ramsey's test procedure for conditionals into a test semantics for such constructions.[3] To get the basic picture in place—and to get a sense for where we need to do better—consider a propositional language to which we have added a special conditional connective ($>$). In a dynamic setting, semantic values are update operations on a carrier of information—for current purposes, a set of possible worlds. The following entries for atomic sentences, negation, and conjunction are the classical ones from Veltman 1996:

(i) $s[p] = \{w \in s\colon w(p) = 1\}$

(ii) $s[\neg\phi] = s \backslash s[\phi]$

(iii) $s[\phi \wedge \psi] = s[\phi] \cap s[\psi]$

An update of an information carrier s with an atomic sentence p eliminates all possible worlds from s at which p is false. Negation is set subtraction while conjunction is set intersection.[4]

It is then straightforward to translate Ramsey's acceptance conditions for conditionals into a dynamic update procedure (see, e.g., Veltman 1985 and Gillies 2004). Say that s accepts ϕ, $s \Vdash \phi$, just in case $s[\phi] = s$, that is, just in case updating s with ϕ idles. Define:

(iv) $s[\phi > \psi] = \{w \in s\colon s[\phi] \Vdash \psi\}$

A conditional tests whether its consequent is accepted once the input state is strengthened with the antecedent. If it is, then the test is passed and returns the original input; if not, the test fails and returns the absurd state ($\varnothing$).

The proposal sketched here is the standard test semantics for conditionals in the dynamic literature, but its key Ramseyan ideas are also prominent in other semantic settings. Yalcin (2007), for instance, proposes that a conditional is true given some world

[3] The classical sources of inspiration for a dynamic approach to semantic theorizing: Discourse Representation Theory (Kamp 1981; Kamp and Reyle 1993; Kamp et al. 2011), Dynamic Predicate Logic (Groenendijk and Stokhof 1991), File Change Semantics (Heim 1982), Update Semantics (Veltman 1985, 1996). The path pursued here is closest in spirit to Update Semantics.

[4] Conjunction is commonly modeled as sequential updating in dynamic semantics—see, for instance, von Fintel and Gillies 2008 and Willer 2015b—but here we do not need to worry about internal dynamics, and so a simple intersective approach will do.

w and state of information s just in case s accepts its consequent once updated with the antecedent (see also Kolodny and MacFarlane 2010). Relatedly, Kratzer (1986, 1991, 2012) proposes that *if*-clauses are restrictors and that in the case of plain indicatives, what is restricted is an (implicit) epistemic necessity modal. On this view a plain conditional is, plus or minus a bit, true at a world of evaluation if the result of restricting the epistemic possibilities with the antecedent entails the consequent. What these views have in common, despite important differences, is the idea that a conditional is acceptable insofar as its consequent is entailed by the result of updating some salient information carrier with the conditional antecedent.

And it does not take much to see that this kind of approach will run into the kind of problems with negation that fuel so much excitement for CEM. Simply note that the semantics for negation and the conditional connective deliver the following result for negated *ifs*:

$$s[\neg(\phi > \psi)] = \{w \in s \colon s[\phi] \nVdash \psi\}$$

For a negated conditional $\ulcorner\neg(\phi > \psi)\urcorner$ to be accepted, it suffices that $s[\phi]$ fails to accept ψ. Accepting a conditional negation $\ulcorner\phi > \neg\psi\urcorner$, in contrast, requires $s[\phi]$ to accept $\ulcorner\neg\psi\urcorner$.

$$s[\phi > \neg\psi] = \{w \in s \colon s[\phi] \Vdash \neg\psi\}$$

Since a state may fail to accept ψ without accepting the negation of ψ—think about taking an agnostic stance on the issue of whether ψ is true—we do not predict that negated conditionals amount to conditional negations. And again, this result is not peculiar to a dynamic setting but is shared by other implementations of Ramsey's basic insight into how conditionals are to be evaluated.

To say that conditionals require their consequent to be accepted, or entailed, by some distinguished state of information once strengthened with the antecedent makes good intuitive sense but does not by itself offer a compelling story about negated conditionals. Williamson (1988) suggests that those who hear negated conditionals as conditional negations are victims of a scope confusion. This is an attractive position insofar as it leaves nothing to worry about, but accepting that ordinary speakers are wrong on such a large scale—recall the variety of data that are at stake here—should strike us as something like a last resort, and in any case one would like to know why conditionals in particular give rise to scope confusions when we have no trouble distinguishing, say, "not necessarily" from "necessarily not." So I say we are well advised to see what can be done about the trouble with negation.

One interesting—though, as I shall indicate below, wanting—strategy proceeds by strengthening the acceptance conditions of conditionals. Cariani and Goldstein (forthcoming) embellish the semantics of conditionals with a homogeneity presupposition. On this proposal, a conditional presupposes that either all antecedent-verifying worlds are consequent-verifying worlds, or that all antecedent-verifying worlds are consequent-falsifying worlds. It is straightforward to implement this idea in our framework by modifying the update rules for conditionals as follows:

(iv′) $$s[\phi > \psi] = \begin{cases} \{w \in s \colon s[\phi] \Vdash \psi\} & \text{if } s[\phi] \Vdash \psi \text{ or } s[\phi] \Vdash \neg\psi \\ \text{undefined} & \text{otherwise} \end{cases}$$

Given minimal assumptions about how the possibility of presupposition failures interacts with logical consequence, the fact that negated conditionals entail conditional negations follows right away. In line with standard protocol, we think of logical consequence as guaranteed preservation of commitment and take presupposition failures as constraining the states of information that we have to consider in evaluating an argument for validity (cf. Beaver 2001):

> $\phi_1, \ldots, \phi_n$ entails ψ, $\phi_1, \ldots, \phi_n \vDash \psi$, iff for all s such that $s[\phi_1]\ldots[\phi_n][\psi]$ is defined, $s[\phi_1]\ldots[\phi_n] \Vdash \psi$.

In a presupposition-free fragment, the above definition amounts to standard update-to-test consequence: check whether every state, once updated with the premises, is committed to the conclusion. If presuppositions are at play, we simply set aside those states for which updating with the premises and then with the conclusion is undefined. Assuming that conditionals presuppose homogeneity, CEM and thus the fact that negated conditionals entail conditional negations follow by design, since it is in the very nature of a CEM counterexample state that it fails to be homogeneous in the relevant way.

The problem with the strategy outlined in the previous paragraph is that there is little reason to think that conditionals do in fact presuppose homogeneity, since people have no trouble evaluating conditionals even if the antecedent fails to settle the question of the consequent in one way or the other. Consider:

(5) If Maria flipped the coin, it landed tails.

Even if all you know is that the coin is fair, it makes perfect sense to consider (5), and in fact it makes sense to assign the sentence .5 probability. This is not at all what we would expect if (5) suffered from a presupposition failure of any kind: for instance, neither (6a) nor (6b) would be assigned positive credence if Maria never smoked or if she only read half of the books, respectively (cf. Santorio 2017).

(6) a. Maria stopped smoking.
b. Maria read the books.

In brief, there is little reason to think that anything resembling homogeneity is true for conditionals, and hence no reason to think that homogeneity is somehow presupposed when we are engaging in conditional discourse and reasoning.

The good news is that we do not really have to tinker with the acceptance conditions of conditionals to deal with the negation problem.[5] Here is the basic idea. Ramsey's account, recall, states what it takes to *accept* a conditional: do so if you hypothetically accept the consequent under the assumption of the antecedent. Such a proposal, of course, also implies what it takes to *fail* to accept a conditional: do so if you fail to hypothetically accept the consequent under the assumption of the antecedent. But it does not trivially state what it takes to *reject* a conditional, for the plain reason that

[5] I set aside a proposal that would draw inspiration from Stalnaker's (1968) uniqueness assumption for conditional selection functions and stipulate that conditional consequents are evaluated against a singleton set of possible worlds. While this move predicts that negated conditionals are equivalent with conditional negations and Stalnaker explicitly takes his proposal to be inspired by the Ramsey test, there is simply no reason to think that assuming conditional antecedents is guaranteed to result in a maximally opinionated state of information.

failing to accept a conditional is not trivially the same as rejecting it. It is exactly this omission that I suggest leaves room for an implementation of the Ramsey test for conditionals that lives happily with everything we have said about negated conditionals. Specifically, I suggest that Ramsey's dictum about accepting conditionals is compatible with the following proposal about what it takes to *reject* a conditional: do so if you (hypothetically) reject the consequent under the assumption of the antecedent. The first step toward getting our story about conditionals straight, then, is to realize that such a story must distinguish between the notion of acceptance ($\Vdash^+$) and the one of rejection ($\Vdash^-$), and we will say:

$$s \Vdash^+ \phi > \psi \text{ just in case } s[\phi] \Vdash^+ \psi$$
$$s \Vdash^- \phi > \psi \text{ just in case } s[\phi] \Vdash^- \psi$$

The first clause simply repeats Ramsey's original acceptance conditions for conditionals; the second adds the earlier proposed rejection conditions for conditionals.

The next step is to explicitly connect the meaning of negation with the notion of rejection. To accept a negation is to reject what is negated; and to reject a negation is to accept what is negated. Precisely:

$$s \Vdash^+ \neg\phi \text{ just in case } s \Vdash^- \phi$$
$$s \Vdash^- \neg\phi \text{ just in case } s \Vdash^+ \phi$$

Even without elaborating this proposal further—and I will—one can already see how it bridges the gap between negating a conditional and committing to a conditional negation: to do the former is to reject (and not just fail to accept) a conditional, and rejecting a conditional amounts to accepting the negation of its consequent under the assumption of its antecedent. This is just what is needed to make sense of negated conditionals.[6]

And we can say all of this while leaving room for there to be genuine indecision as to whether to accept a conditional or its negation. Whenever s accepts neither ψ nor its negation if strengthened with ϕ, neither $\ulcorner \phi > \psi \urcorner$ nor $\ulcorner \neg(\phi > \psi) \urcorner$ are accepted in s—nor is $\ulcorner \phi > \neg\psi \urcorner$, and so we have effectively disentangled the problem of negated conditionals from the issue of CEM: one can correctly predict that negated conditionals entail conditional negations without subscribing to conditional excluded middles.

2.2 Details

Let me briefly state in more detail how the embellished Ramseyan proposal can be implemented in a dynamic setting. The target language $\mathcal{L}$ is the smallest set that contains a set of propositional atoms $\mathcal{A} = \{p, q, r, \ldots\}$, and is closed under negation ($\neg$), conjunction ($\wedge$), disjunction ($\vee$), and the Ramsey conditional ($>$), though to streamline the discussion I shall assume that conditional antecedents are selected from the classical fragment $\mathcal{L}_0$ of $\mathcal{L}$.[7] Just as we distinguished between the attitudes of acceptance and rejection, we

[6] Groenendijk and Roelofsen (2015) as well as Ciardelli (forthcoming) outline approaches to (negated) conditionals that are similar in spirit to the one pursued here, though they do not discuss the significance of these approaches for the familiar narrative on conditionals and Conditional Excluded Middle.

[7] Thus I set aside the possibility of the Ramsey conditional appearing in conditional antecedents. This assumption is not essential for the story told here to make good sense but streamlines some of its more technical aspects.

may distinguish between a positive, acceptance-inducing update function $[\cdot]^+$ and a negative, rejection-inducing update function $[\cdot]^-$. Start with the obvious entries for atomic sentences and negation.

$$(\mathcal{A}) \quad \begin{array}{l} s[p]^+ = \{w \in s\colon w(p) = 1\} \\ s[p]^- = \{w \in s\colon w(p) = 0\} \end{array} \qquad (\neg) \quad \begin{array}{l} s[\neg\phi]^+ = s[\phi]^- \\ s[\neg\phi]^- = s[\phi]^+ \end{array}$$

A positive update with p eliminates from the input state all possible worlds at which p is true, while a negative update with p eliminates all possible worlds at which p is false. A positive update with $\ulcorner\neg\phi\urcorner$ is a negative update with ϕ, and a negative update with $\ulcorner\neg\phi\urcorner$ is a positive update with ϕ.

To run a positive update with a conjunction, we (i) update the input state with the first conjunct, and then (ii) intersect the output with the result of updating the input state with the second conjunct; a negative update with a conjunction is then the union of (i) the result of updating with the negation of the first conjunct and (ii) the result of updating with the negation of the second conjunct:

$$(\wedge) \quad \begin{array}{l} s[\phi \wedge \psi]^+ = s[\phi]^+ \cap [\psi]^+ \\ s[\phi \wedge \psi]^- = s[\phi]^- \cup [\psi]^- \end{array}$$

Disjunction may then receive its standard definition in terms of negation and conjunction.

Conditionals remain tests but we now translate both acceptance and rejection conditions into a dynamic setting. Say that s accepts ϕ, $s \Vdash^+ \phi$, just in case $s[\phi]^+ = s$ and that s rejects ϕ, $s \Vdash^- \phi$, just in case $s[\phi]^- = s$. The basic proposal would then be that a conditional of the form $\ulcorner\phi > \psi\urcorner$ is accepted by s just in case $s[\phi]^+ \Vdash^+ \psi$ and rejected just in case $s[\phi]^+ \Vdash^- \psi$. One minor wrinkle: following standard protocol (see, e.g., von Fintel 2001, Gillies 2007, Willer 2017) we shall assume that conditionals presuppose that their antecedent be compatible with the relevant modal domain and since—for now anyway—conditionals impose tests on the input context s, this is just to presuppose that the conditional antecedent is a possibility in the input state. Putting all of this together, we say:

$$(>) \quad \begin{array}{l} s[\phi > \psi]^+ = \{w \in s\colon s[\phi]^+ \Vdash^+ \psi\} \text{ iff } s[\phi]^+ \neq \varnothing \\ s[\phi > \psi]^- = \{w \in s\colon s[\phi]^+ \Vdash^- \psi\} \text{ iff } s[\phi]^+ \neq \varnothing \end{array}$$

Positive and negative updates with conditionals both require that their antecedent be compatible with the input state. If defined, a positive update with a conditional tests whether its consequent is accepted once the input state is strengthened with the antecedent. If defined, a negative update with a conditional tests whether its consequent is rejected once the input state is strengthened with the antecedent. As before, a passed test returns the original state; a failed test results in the absurd state ($\varnothing$).

Bilateral setups such as the one we are exploring here allow for a variety of notions of logical consequence, and we will exploit this flexibility momentarily (and use subscripts to keep track of the options). For now, one way to go is to think of valid inferences as those whose premises induce acceptance of the conclusion (assuming definedness).

> A sequence $\phi_1, \ldots, \phi_n$ *induces acceptance* of ψ, $\phi_1, \ldots, \phi_n \vDash_1 \psi$, just in case for all s such that $s[\phi_1]^+ \ldots [\phi_n]^+[\psi]^+$ is defined, $s[\phi_1]^+ \ldots [\phi_n]^+ \Vdash^+ \psi$.

Thinking of entailment as acceptance inducing is to recreate update-to-test consequence in a bilateral setting. A special case: say that a state s *admits* ϕ just in case updating s with ϕ (positively or negatively) is defined. Then ϕ is a validity just in case every state that admits ϕ also accepts ϕ.

The resulting framework then predicts that negated conditionals entail conditional negations (and vice versa):

Fact 1 $\neg(\phi > \psi) \vDash_1 \phi > \neg\psi$

This is easy to see since every positive update with $\ulcorner\neg(\phi > \psi)\urcorner$ is a negative update with $\ulcorner\phi > \psi\urcorner$, which is testing whether ψ is rejected—and thus whether $\ulcorner\neg\psi\urcorner$ is accepted—under the assumption of ϕ. But this is just what $\ulcorner\phi > \neg\psi\urcorner$ is asking, which establishes the fact.

At the same time, CEM turns out to be invalid if validity amounts to guaranteed acceptance by every admitting state:

Fact 2 $\nvDash_1 (\phi > \psi) \vee (\phi > \neg\psi)$

A very simple counterexample is a state that accepts p but is agnostic about q, say $s = \{w_1, w_2\}$ such that $w_1(p) = w_2(p) = 1$ and $w_1(q) = 1$ while $w_2(q) = 0$. Then $s[p]^+ = s$ but clearly $s \not\Vdash^+ q$ and $s \not\Vdash^- q$ and hence $s[p > q]^+ = s[p > \neg q]^+ = \varnothing$. So $s[(p > q) \vee (p > \neg q)]^+ = \varnothing$ and since s is not empty we have $s[(p > q) \vee (p > \neg q)]^+ \neq s$ and so $s \not\Vdash^+ (p > q) \vee (p > \neg q)$, which establishes the fact.

This concludes the first key lesson of the current exercise: that there need not be a trade-off between signing up for a universalist semantics that rejects Conditional Excluded Middle and accounting for how conditionals play with negation in natural language. What allows us to see that—if we adopt a Ramseyan perspective anyway—is the realization that an account of what it takes to *accept* a conditional lives happily with the existence of a separate characterization of what it takes to *reject* a conditional. Once we couch our analysis of conditionals in a *bilateral* setting that is grounded in two primitive basic semantic concepts—truth and falsity, acceptance and rejection, or positive and negative updating—it becomes straightforward to see that the problem with negating conditionals is really about getting the meaning of *negation* straight rather than coming up with a theory of conditionals that validates CEM.

Bilateral stories like the one told here inevitably raise the spectre of the Frege-Geach problem. Textbook metaethical expressivism starts with the natural intuition that a sentence like (7a) "Stealing is wrong" is used to express the attitude of disapproval of stealing.[8] Natural as this may sound, it raises the question of what attitude the negation of (7a), i.e. (7b), expresses. Consider:

(7) a. Stealing is wrong.
b. Stealing is not wrong.
c. Not stealing is wrong.

The initial observation here is that (7b) cannot express disapproval of not stealing, since this is what (7c) expresses; nor can it simply express the absence of the attitude of

[8] See Schroeder 2010 and references therein for a discussion of metaethical expressivism and its history.

disapproval of stealing, since someone who is thoroughly agnostic or undecided about the moral status of stealing fails to disapprove of stealing but does not endorse (7b). The obvious conclusion to draw here is that an expressivist analysis of the language of morals must appeal to a second basic attitude besides disapproval—say, the attitude of *tolerance*—and that (7b) expresses that very attitude toward stealing.

The challenge then is to explain why (7a) and (7b) seem to express *incompatible* attitudes. This is especially pressing for expressivists, who aim at explaining the inconsistency of sentences in terms of an incompatibility between the states of mind expressed by these sentences: so (7a) and (7b) must be inconsistent *because* they express incompatible states of mind. But the issue is perfectly general, and the problem, as Schroeder (2008) explains, is that all that the story sketched so far delivers is that (7a) and (7b) express distinct attitudes toward the same kind of action that—as far as the analysis is concerned—might very well be logically unrelated. If all we have are two basic attitudes—tolerance and disapproval—the incompatibility between the states of mind expressed by (7a) and (7b) remains a matter of pure stipulation.

Some have insisted that there is nothing wrong with grounding one's semantics in a primitive notion of incompatibility between basic attitudes (see e.g. Gibbard 2013). My goal here is not to referee this issue or to dive into a detailed discussion of the language of morals but simply to highlight that the current framework has no need for such maneuvers. It would, on first and second sight anyway, be dissatisfying if we had to *stipulate* that no single state of information can both accept and reject a single sentence at the same time. Fortunately, however, it is straightforward to verify that the attitudes of acceptance and rejection are incompatible in the following sense:

Fact 3 For all s and ϕ: if $s \Vdash^{+} \phi$ and $s \Vdash^{-} \phi$, then $s = \varnothing$.[9]

No non-absurd state, in other words, can both accept and reject a sentence. For instance, suppose that s is non-absurd and accepts the conditional "If Mary is in Chicago, then Jack is in Rome" ($c > r$). Then $s[c]^{+}$ is a non-absurd state and, moreover, $s[c]^{+} \Vdash^{+} r$. Since no non-absurd state can both accept and reject that Jack is in Rome (that is, exclusively consist of r-worlds and of $\neg r$-worlds), it follows that $s[c]^{+} \nVdash^{-} r$ and so that $s \nVdash^{-} c > r$, that is, s fails to reject "If Mary is in Chicago, then Jack is in Rome." For parallel reasons, if s is non-absurd and rejects "If Mary is in Chicago, then Jack is in Rome," then the state must fail to accept the conditional.

The previous result depends, of course, on the details of the semantics of negation: we *could* have provided negative entries for some or all of our connectives (or atomic sentences) that make the attitudes of acceptance and rejection compatible with each other. Nonetheless, the fact remains that there is nothing *per se* dubious about a framework that appeals to two primitive semantic notions in semantic theorizing, and that the incompatibility between the attitudes of acceptance and rejection is not a mere matter of stipulation but something that follows from our semantics. Let us explore the framework a bit further.

[9]The key underlying observation here is that if defined, the intersection of $s[\phi]^{+}$ and $s[\phi]^{-}$ is guaranteed to be empty (the proof is straightforward via induction). But if $s \Vdash^{+} \phi$ and $s \Vdash^{-} \phi$, then $s[\phi]^{+} = s[\phi]^{-} = s[\phi]^{+} \cap s[\phi]^{-} = s$, and so it follows that s must be the absurd state.

3 Conditional Excluded Middle for the Ramsey Test

While the story told so far has taught us something important about conditionals—namely, that the challenge of validating CONDITIONAL EXCLUDED MIDDLE is separate from the task of explaining why negated conditionals seem to entail conditional negations—there is at least some reason to think that more needs to be said about the interplay between *if*, *or*, and *not*. To see this, consider again the case of a perfectly fair coin flip:

(8) a. If Maria flipped the coin, it landed tails.
 b. If Maria flipped the coin, it landed heads.

The simple intuition here is that it makes good sense to assign .5 probability to (8a) and .5 probability to (8b); since (8a) and (8b) are mutually exclusive, their disjunction has a probability of 1 and thus the distinct air of a validity after all (cf. Santorio 2017). So while the empirical data about negated conditionals do nothing to demonstrate that CEM is valid, there are other considerations that give it some intuitive appeal. Whether these are, at the end of the day, irresistible is a question that must be left to another day.[10] Here I will show how to accommodate the possibility that CEM is valid, and I will do so by exploiting the fact (already indicated earlier) that a bilateral semantics provides us with some interesting options for how to think about logical consequence.

Start again with the difference between acceptance failures and rejection. Earlier we saw why a disjunction like "$(p > q) \vee (p > \neg q)$" need not be accepted by an information carrier: if one is certain about p but agnostic about q, one would not accept "$p > q$" and one would not accept "$p > \neg q$," and so the whole disjunction would not be accepted either. But again failing to accept ϕ is not the same as rejecting ϕ, and in particular we can observe that while a state s need not accept "$(p > q) \vee (p > \neg q)$," it cannot reject it either. For doing so requires that $s[(p > q) \vee (p > \neg q)]^- = s$ and so $s[p > q]^-[p > \neg q]^- = s$, which in turn requires that s must reject both "$p > q$" and "$p > \neg q$." And that, in turn, requires that $s[p]^+$ is consistent but rejects both q and its negation, which simply cannot happen. The case, indeed, generalizes:

Fact 4 For all s, ϕ and ψ: $s\big[(\phi > \psi) \vee (\phi > \neg\psi)\big]^- = \varnothing$.

This is an immediate consequence of the fact that no non-absurd state can accept both ϕ and its negation (recall Fact 3).

The previous fact is suggestive, since it highlights an alternative way of thinking about logical consequence and thus of validities. Earlier we suggested that an argument is valid just in case updating with the premises rationally commits one to accepting the conclusion; but instead we may think of an argument as valid just in case one could rationally update with the premises and also reject the conclusion. More precisely:

> A sequence $\phi_1, \ldots, \phi_n$ *excludes rejection* of ψ, $\phi_1, \ldots, \phi_n \vDash_2 \psi$, just in case for all s such that $s[\phi_1]^+ \ldots [\phi_n]^+[\psi]^+$ is defined, $s[\phi_1]^+ \ldots [\phi_n]^+[\psi]^- \Vdash \bot$.

[10] I must set aside for now a comprehensive discussion of how credences are assigned to conditionals given a dynamic semantic analysis. But see Ciardelli forthcoming for a proposal that fits well with the spirit of the framework developed here.

Here "$\bot$" represents an arbitrary contradiction. This proposal then treats ϕ as a validity just in case $s[\phi]^- = \varnothing$ for all states that admit ϕ.

In a classical setup—and even when it comes to update-to-test consequence—it does not matter whether we say that the premises of an argument induce acceptance of a conclusion or that they exclude its rejection, for whenever $\phi_1, \ldots, \phi_n$ entails ψ, then $\phi_1, \ldots, \phi_n, \neg\psi$ entails $\bot$, and vice versa. But it does matter in the bilateral setting that is explored here. Specifically, it follows immediately from Fact 4 that Conditional Excluded Middle is valid if entailment requires that updating with the premises exclude rejection of the conclusion.

Fact 5 $\models_2 (\phi > \psi) \vee (\phi > \neg\psi)$

If no consistent state can accept ψ and its negation, it cannot reject ψ and its negation either. So in particular, if consistent, $s[\phi]^+$ cannot reject ψ and its negation, and so (if defined in the first place) $s[\phi > \psi]^- = \varnothing$ or $s[\phi > \neg\psi]^- = \varnothing$.

Validity understood as exclusion of rejection ("validity$_2$" for short) is thus an interesting alternative to the more familiar concept of validity as guaranteeing acceptance ("validity$_1$" for short).[11] It also preserves the earlier made prediction about the interplay between negated conditionals and conditional negations, since whenever an argument is valid$_1$, it is also valid$_2$:

Fact 6 If $\phi_1, \ldots, \phi_n \models_1 \psi$, then $\phi_1, \ldots, \phi_n \models_2 \psi$.

No state that accepts ψ can be updated with the negation of ψ without resulting in the absurd state. So if a sequence $\phi_1, \ldots, \phi_n$ guarantees acceptance of ψ, then it also excludes rejection of ψ. Thus validity$_2$ preserves what we want about negated conditionals and, in addition, delivers CEM as valid.

The second important upshot of this discussion, then, is that Conditional Excluded Middle lives happily in a framework that treats conditionals as universal quantifiers over a contextually determined set of possible worlds. The key idea here, again, is that rejecting a conditional is not the same as just failing to accept it. This, recall, allowed us to get the facts about negated conditionals straight without endorsing CEM. But it also turns out that while not every instance of CEM needs to be accepted, all of them are resistant to rejection, and this inspires a notion of validity as excluded rejection that delivers CEM as a tautology after all. A bit more can be said about the intuitive underpinning of this notion of entailment; before that, let me demonstrate that it has a few additional notable consequences.

Conditional Excluded Middle is not the only principle that, while having something going for it, fails to hold in a classical analysis of conditionals as strict quantifiers over some contextually determined set of possible worlds. Another is Simplification of Disjunctive Antecedents (SDA):

$$\text{(SDA)} \qquad (\phi \vee \psi) > \chi \models (\phi > \chi) \wedge (\psi > \chi)$$

[11] This is not to say that validity$_2$ is the only notion of logical consequence that delivers reductio failures. In Update Semantics, test-to-test consequence violates reductio reasoning; so does the notion of informational consequence in Yalcin 2007.

While the case for CEM tends to be indirect, SDA has the immediate air of an intuitive validity:

(9) If you pay cash or with a debit card, you will receive a five percent discount.
$\rightsquigarrow$ If you pay cash, you will receive a five percent discount.
$\rightsquigarrow$ If you pay with a debit card, you will receive a five percent discount.

Intuitively, (9) suggests that you get five percent if you pay cash, *and* if you pay with a debit card. This is not an easy prediction to make and it may also, as Cariani and Goldstein (forthcoming) show, carry substantial trouble in its wake if one simultaneously wants to validate CEM. Nonetheless the story told here gets us a far way toward a safe passage toward endorsing both principles.

Let me begin with the observation that SDA is in fact valid if we understand logical consequence as exclusion of rejection.

Fact 7 $(\phi \vee \psi) > \chi \vDash_2 (\phi > \chi) \wedge (\psi > \chi)$

The underlying fact here is that if ψ is accepted in some state s, then ψ it is not rejected by any consistent strengthened state $s' \subseteq s$. So if $s[\phi \vee \psi]^+ \Vdash^+ \chi$, then both $s[\phi]^+ \nVdash^- \chi$ and $s[\psi]^+ \nVdash^- \chi$. It follows that neither $\ulcorner \phi > \chi \urcorner$ nor $\ulcorner \psi > \chi \urcorner$ can be rejected by s.

Cariani and Goldstein (forthcoming) show that, taken together, CEM and SDA have the potential of bringing a number of unwelcome consequences in their wake, given minimal additional assumptions. One of them is that the following principle turns out to be a validity, which Cariani and Goldstein label the Interconnectivity of All Things (IAT):

$$\text{(IAT)} \qquad \vDash ((\phi > \chi) \wedge (\psi > \chi)) \vee ((\phi > \neg\chi) \wedge (\psi > \neg\chi))$$

What makes IAT so unattractive is that it clashes with the uncontroversial fact that one may endorse the conjunction of (10a) and (10b):

(10) a. If Maria comes to the party, it will be fun.
b. If Bill comes to the party, it won't be fun.

So IAT is no good. But (to sketch an argument that Cariani and Goldstein lay out in more detail) if CEM is a validity, then so in particular is $\ulcorner((\phi \vee \psi) > \chi) \vee ((\phi \vee \psi) > \neg\chi)\urcorner$. And if such conditionals simplify—as they should if SDA is really valid—it seems to follow right-away that IAT is a validity as well.

And yet it is easy to verify that IAT is not a validity in the bilateral system that we have been exploring so far. Consider the instance "$((p > r) \wedge (q > r)) \vee ((p > \neg r) \wedge (q > \neg r))$" and let $s = \{w_1, w_2\}$ with the following distribution of truth-values:

	p	q	r
w_1	T	F	F
w_2	F	T	T

It is easy to see that $s \Vdash^+ p > \neg r$ and so $s \Vdash^- p > r$ and, moreover, $s \Vdash^+ q > r$ and so $s \Vdash^- q > \neg r$. Accordingly, $s[(p > r) \wedge (q > r)]^- = s$ and $s[(p > \neg r) \wedge (q > \neg r)]^- = s$ and so $s \Vdash^- ((p > r) \wedge (q > r)) \vee ((p > \neg r) \wedge (q > \neg r))$.

This is a bit of a surprising result: if CEM is valid and conditionals of all stripes simplify, how could IAT not be a validity as well? The answer is that validity$_2$ fails to be transitive.

Fact 8 For some ϕ, ψ, and χ: $\phi \vDash_2 \psi$ and $\psi \vDash_2 \chi$ but $\phi \nvDash_2 \chi$.

Specifically, we have, with "$\top$" being any tautology: (i) $\top \vDash_2 ((p \vee q) > r) \vee ((p \vee q) > \neg r)$ and (ii) $((p \vee q) > r) \vee ((p \vee q) > \neg r) \vDash_2 ((p > r) \wedge (q > r)) \vee ((p > \neg r) \wedge (q > \neg r))$, but also (iii) $\top \nvDash_2 ((p > r) \wedge (q > r)) \vee ((p > \neg r) \wedge (q > \neg r))$. The reason: (i) says that no state may consistently reject a certain CEM-instance and (ii) says that once some CEM-instance is accepted, the corresponding IAT instance cannot be rejected. But just because s cannot consistently reject ϕ does not mean that s must accept ϕ. Accordingly, just because some CEM-instance cannot be rejected does not mean that it must be accepted, and so it is also perfectly possible for certain IAT-instances to be rejected by a non-absurd state of information.

Cariani and Goldstein also present a special problem for dynamic analyses of conditionals. Consider the following semantic entries for the necessity operator ($\Box$):

$$s[\Box\phi]^+ = \{w \in s \colon s \Vdash^+ \phi\}$$
$$s[\Box\phi]^- = \{w \in s \colon s \nVdash^+ \phi\}$$

A positive update with $\ulcorner\Box\phi\urcorner$ checks whether ϕ is accepted, while a negative update checks whether whether ϕ fails to be accepted. It follows immediately:

Fact 9 $\phi > \psi \vDash_2 \Box(\neg\phi \vee \psi)$

That is just to say that there is a distinct sense in which the Ramsey conditional is a strict material conditional. The worry about this is that if CEM is valid, then $\ulcorner\Box(\neg\phi \vee \psi) \vee \Box(\neg\phi \vee \neg\psi)\urcorner$ should be valid in the present system as well. And that would be bad news, for it would mean that, for instance, "$\Box(\neg\top \vee p) \vee \Box(\neg\top \vee \neg p)$" and thus also "$\Box p \vee \Box\neg p$" have the honor of being a validity. And yet these require that every state either accepts p or its negation, which would make our information carriers way too opinionated (in fact, every consistent information carrier would contract into a singleton set).

Again, the problem is unreal: there is nothing wrong with a state that fails to settle every possible question in one way or another. And again the triviality argument fails to go through since the notion of validity at play in the system developed here fails to be transitive: yes, CEM is valid and once one accepts a given CEM-instance one cannot reject the corresponding disjunction of strict material conditionals. And yet the latter may very fail to be a validity—may very well be rejected by a consistent state—for the already familiar reason that excluded rejection is not the same as guaranteed acceptance.

Thinking of validity as exclusion of rejection, then, shows that Conditional Excluded Middle can live happily in Ramseyan setting, and that it can co-exist with Simplification of Disjunctive Antecedents without negative side effects. But before I tie up some loose ends, it is useful to ask whether validity$_2$ has any intuitive foundation.

I do not have that much to say in defense of transitivity failures: finding a notion of logical consequence that exhibits this phenomenon requires, for sure, roaming in non-classical territory, but not necessarily in dark corners of logical space. Applying the popular notion of STRAWSON ENTAILMENT (Strawson 1952, von Fintel 1997, 1999, 2001) to the language of presupposition, for instance, immediately brings transitivity failures in its wake:

> A sequence $\phi_1, \ldots, \phi_n$ *Strawson entails* ψ just in case $\phi_1, \ldots, \phi_n, \phi_{n+1}$ classically entails ψ, where ϕ_{n+1} is a premise stating that the presuppositions of all the statements involved are satisfied.

Combined with Strawson's (1950) analysis of definite descriptions as presupposing existence and uniqueness, (11a) is Strawson valid and entails (11b), and yet (11b) is not Strawson valid:

(11) a. The king of France is bald or not bald.
b. There is a unique King of France.

Similarly, transitivity will fail once we combine Stalnaker's (1975) concept of a reasonable inference with an analysis of indicative conditionals as presupposing the openness of their antecedent in context (see Cariani and Goldstein forthcoming for discussion). Thus the particular feature of validity$_2$ that allows us to maintain both CEM and SDA should not strike us as especially uncanny.

Perhaps more importantly, there is no major gulf between the two notions of logical consequence that have been at the center of attention of the current discussion. Here the key thought is that validity$_1$ and validity$_2$ coincide over a distinguished set of information states, namely those that obey the law of excluded middle with respect to the consequent under the assumption of the premises. The key point here is the following:

Fact 10 For all $s \subseteq W$: if $s[\phi]^+ \ldots [\phi_n]^+ \Vdash^+ \psi \vee \neg\psi$, then $s[\phi]^+ \ldots [\phi_n]^+ \Vdash^+ \psi$ iff $s[\phi]^+ \ldots [\phi_n]^+[\psi]^- \Vdash^+ \bot$.

This matters since, as we noted earlier, entailment as guaranteed acceptance already sets aside those states for which updating with the premises $\phi_1, \ldots, \phi_n$ and then with the conclusion ψ is undefined; if we now slightly strengthen the requirement so that $s[\phi]^+ \ldots [\phi_n]^+$ is not only defined but also accepts $\ulcorner\psi \vee \neg\psi\urcorner$ (which entails but goes beyond asking that $s[\phi]^+ \ldots [\phi_n]^+[\psi]^+$ is defined) we arrive at the notion of entailment as excluded rejection. I thus submit that there is a smooth transition from the well-established notion of entailment as guaranteed acceptance to the less familiar but highly interesting conception of entailment as excluded rejection.

4 Loose Ends

I will first offer some remarks on an alternative path to CEM that also threatens to trivialize some of the crucial distinctions that I have previously drawn (Section 4.1) and then address the issue of antecedent strengthening (Section 4.2) .

4.1 Excluded Middle

I have explained why CEM is not required to explain the observation that ordinary speakers have the habit of interpreting negated conditionals as conditional negations. One may insist, however, that CEM does follow from this observation if we just add the Law of Excluded Middle (LEM) to the mix:

$$\text{(LEM)} \qquad \models \phi \vee \neg\phi$$

If negated conditionals entail their corresponding conditional negations, then CEM follows (classically) from LEM. The proposal would then be to add the validity of LEM as an innocent background constraint to the picture, and to derive the validity of CEM from the interaction between negation and conditionals on these grounds.[12]

The argument outlined in the previous paragraph is of genuine interest, since we already saw that validity$_1$ and validity$_2$ collapse in the framework developed here whenever the Law of Excluded Middle holds—so if LEM is valid across the board, the distinction between the two notions of validity turns out to be trivial. The case is not irresistible, however, precisely because LEM cannot be taken for granted, and indeed we expect the law to fail whenever negation is stronger than one would expect in light of the positive form. Consider, for instance, the empirical facts surrounding the free choice effect. On the one hand, the possibility of a disjunction seems to entail the possibility of each disjunct:

(12) We can speak English or Hungarian with each other.
 a. $\rightsquigarrow$ We can speak English with each other.
 b. $\rightsquigarrow$ We can speak Hungarian with each other.

At the same time, embedding a disjunctive possibility under negation does not merely rule out one of the disjuncts as a possibility, but both:

(13) We cannot speak English or Hungarian with each other.
 a. $\rightsquigarrow$ We cannot speak English with each other.
 b. $\rightsquigarrow$ We cannot speak Hungarian with each other.

These facts have been extensively discussed in the literature,[13] and they suggest—on first and second sight anyway—the following two principles:

(FC) $\Diamond(\phi \vee \psi) \models \Diamond\phi \wedge \Diamond\psi$

(DP) $\neg\Diamond(\phi \vee \psi) \models \neg\Diamond\phi \wedge \neg\Diamond\psi$

Taken together, FC and DP put pressure on LEM. Consider a context in which everyone present speaks English but no one present speaks Hungarian. Then all of the following seem false (or not accepted):

(14) a. We can speak English or Hungarian with each other.

[12] This argument from "negation swap" for conditionals to CEM via LEM is inspired by a structurally analogous line of reasoning in the literature on *will*: "*will* not ϕ" and "not *will* ϕ" ring equivalent; together with LEM, this means that "will ϕ or will not ϕ" is a validity, which is bad news for standard modal analyses of *will* (see in particular Cariani forthcoming).

[13] See, for instance, Kamp 1973; Alonso-Ovalle 2006; Fox 2007; Zimmermann 2000.

b. We cannot speak English or Hungarian with each other.

c. We can speak English or Hungarian with each other, or we cannot speak English or Hungarian with each other.

Note here that (14c) is an instance of the LAW OF EXCLUDED MIDDLE.

There is, of course, some room for maneuver here if one really wanted to preserve LEM,[14] but the fact remains that we have good reason not to take its validity simply for granted. So the case for CEM that we have considered at the beginning of this section fails because LEM is not irresistible, and the fact that ordinary speakers have the habit of interpreting negated conditionals as conditional negations continues to be insufficient to establish the validity of CEM.

A remaining question might be why instances of LEM should easily roll off the tongue, even if free choice effects or conditionals are at play. For instance, the following two disjunctions have the air of a validity:

(15) a. Either Mary might be in Chicago or in New York, or not.

b. Either it'll rain if you do a rain dance, or not.

One way to go here is to suggest that (15a) and (15b) involve an unarticulated "truth-operator" with the following semantics:

$$(\text{TRUE}) \qquad \begin{array}{l} s[\text{TRUE } \phi]^+ = \{w \in s : w \in s[\phi]^+\} \\ s[\text{TRUE } \phi]^- = \{w \in s : w \notin s[\phi]^+\} \end{array}$$

So (15a) and (15b) sound valid because we hear them as suggesting that the first disjunct is true or not true in the sense specified above, and indeed it is straightforward to verify that thus interpreted the sentences are indeed tautologous:

Fact 11 $\vDash_1 \text{TRUE } \phi \vee \neg\text{TRUE } \phi$

So while LEM is not 1-valid, something close in its vicinity is.

A truth-operator like the one proposed here is useful to explain cases like the following:

(16) It's not true that if you're over 65, you qualify for a discount—you also have to be a veteran.

The denial in (16) seems to target the law-like connection between antecedent and consequent without committing to there being a law-like connection between antecedent and the negation of the consequent. Modeling (16) as an instance of $\ulcorner\neg\text{TRUE}(\phi > \psi)\urcorner$ would explain why this is so.

[14] For instance, one may choose a homogeneity based approach to free choice effects à la Goldstein (forthcoming); if homogeneity is a presupposition, and given a suitable notion of logical consequence, cases like (14c) would not qualify as counterexamples to LEM.

4.2 Antecedent Strengthening

The story told so far gets us a long way toward a powerful framework for conditionals without negative side effects, but it still suffers from a problem that has been traditionally associated with the strict analysis of conditionals in that it validates Antecedent Strengthening (AS):

$$\text{(AS)} \qquad \phi > \chi \vDash (\phi \wedge \psi) > \chi$$

Lewis (1973) notes that AS is undesirable since *Sobel sequences* appear to be perfectly consistent:

(17) If Alice had come to the party, it would have been fun. But if Alice and Bert had come to the party, it would not have been fun.

(17) is a sequence of counterfactuals but the point applies to conditionals of all stripes (see e.g. Willer 2017). And yet it is easy to see that no state that treats ϕ and ψ as possibilities can accept χ if strengthened with ϕ but reject χ if strengthened with ϕ and ψ.

The issue is familiar and so is its solution: conditional antecedents may bring hitherto ignored possibilities into view and so while *ifs* are universal quantifiers over a set of possible worlds, this domain evolves dynamically as discourse proceeds (see von Fintel 2001 and Gillies 2007 for seminal discussion). So in entertaining the possibility of Alice coming to the party, we might ignore the possibility of Bert coming as well, and accept that the party will be fun on these grounds. Once we consider a conditional whose antecedent presupposes the possibility of Alice and Bill coming to the party, the modal horizon shifts, and it may very well be that all the possibilities thus brought into view are such that the party is not fun.

The first step is then to make the reasonable suggestion that in an information state, not all possibilities are created equal: some possibilities are more salient than others, and it makes good sense to then model an input state as a set of sets of possible worlds that is ordered by the subset relations (reminiscent of Lewis's (1973) systems of spheres) and that thus keeps track of how possibilities compare in terms of their salience.

> A *complex* information state $\pi \subseteq \mathcal{P}(W)\backslash\{\varnothing\}$ is a (possibly empty) set of non-empty sets of possible worlds such that for all $s, s' \in \pi$, $s \subseteq s'$ or $s' \subseteq s$. The minimal element (or center) of π is defined as $s_\pi = \{w \colon \exists s \in \pi \forall s' \in \pi.\, w \in s \text{ and } s \subseteq s'\}$. We will say that $s \preceq_\pi s'$ iff $s, s' \in \pi$ and $s \subseteq s'$.

The basic idea then is that conditionals quantify over the minimal element of an information state but also may expand that sphere in virtue of the possibility presuppositions of their antecedents.

The obvious continuation of the story is then to say that conditionals presuppose that the truth of their antecedent be compatible with the minimal sphere and that whenever this is not so we expand the modal horizon until a suitable possibility comes into view. However, we now risk losing the validity of Simplification of Disjunctive Antecedents since the truth of a disjunction does not entail the truth of both disjuncts and so the minimal sphere compatible with "$p \vee q$" need not be the minimal sphere compatible with (say) p. So if the possibility of horizon shifts undercuts the validity of AS it should also undercut the validity of SDA. The case in fact is parallel to Lewis's (1973) system in the sense that the same mechanism that blocks AS also blocks SDA. Some extra care is needed.

The key observation in response to this problem concerns what it takes to entertain a disjunctive possibility. One may, for sure, entertain the possibility of Alice coming to the party without Alice and Bert coming to the party. But in entertaining the possibility of Alice or Bert coming to the party, one is entertaining the possibility of Alice coming to the party, and the one of Bert coming to the party. The idea, in brief, then is that a conditional such as (18a) already brings into view the possibilities that matter for the antecedents of (18b) and (18c).

(18) a. If Alice or Bill come to the party, it will be fun.
b. If Alice comes to the party, it will be fun.
c. If Bill comes to the party, it will be fun.

Willer (2015a, 2018) explicitly connects the reasons for why conditionals simplify with the free choice effect for possibility modals. The basic observation, already discussed earlier, is that disjunctive possibilities imply each of their disjuncts as possibilities.

Let me briefly outline how the idea can be implemented (drawing inspiration from Willer 2015a, 2018). As a first step, I will move from update functions to update relations as the basic semantic value, which (unlike their right-unique cousins) may relate an input state to more than one output state. I will also relativize our basic semantic values to another information state that will ultimately matter for the evaluation of modal statements, including conditionals (the MODAL DOMAIN). Still, the entries for our atoms and negation are simple rewrites of what we said earlier.

$$(\mathcal{A}) \quad \begin{array}{l} s[p]^+_i t \text{ iff } t = \{w \in s \colon w(p) = 1\} \\ s[p]^-_i t \text{ iff } t = \{w \in s \colon w(p) = 0\} \end{array} \qquad (\neg) \quad \begin{array}{l} s[\neg\phi]^+_i t \text{ iff } s[\phi]^-_i t \\ s[\neg\phi]^-_i t \text{ iff } s[\phi]^+_i t \end{array}$$

So a state s is positively p-related to t just in case the latter includes all and only of the former's p-worlds; s is negatively p-related to t just in case the latter includes all and only of the former's $\neg p$-worlds. A negation positively relates two states just in case they are negatively related via what is negated; it negatively relates two states just in case they are positively related via what is negated.

Updating some state s now proceeds by taking the union of its update relata, treating s itself as the modal domain, and we re-define the notions of acceptance and rejection on this basis:

$$\begin{array}{llll} \text{(i)} & s \uparrow \phi = \bigcup\{t : s[\phi]^+_s t\} & \text{(ii)} & s \downarrow \phi = \bigcup\{t : s[\phi]^-_s t\} \\ \text{(iii)} & s \Vdash^+ \phi \text{ iff } s \uparrow \phi = s & \text{(iv)} & s \Vdash^- \phi \text{ iff } s \downarrow \phi = s \end{array}$$

Acceptance and rejection are once again understood as fix point properties.

The proposal for conjunction is as follows:

$$(\wedge) \quad \begin{array}{l} s[\phi \wedge \psi]^+_i t \text{ iff } \exists u.\, s[\phi]^+_i u \text{ and } u[\psi]^+_i t \\ s[\phi \wedge \psi]^-_i t \text{ iff } s[\phi]^-_i t \text{ or } s[\psi]^-_i t \end{array}$$

Assuming again that conjunction and disjunction are duals, we arrive at the following entries for the latter connective:

$$(\vee) \quad \begin{array}{l} s[\phi \vee \psi]_i^+ t \text{ iff } s[\phi]_i^+ t \text{ or } s[\psi]_i{}^+ t \\ s[\phi \vee \psi]_i^- t \text{ iff } \exists u.\, s[\phi]_i^- u \text{ and } u[\psi]_i^i t \end{array}$$

Note here that we once again set aside internal dynamic effects: when updating with a conjunction (disjunction), both conjuncts (disjuncts) are processed in light of the same modal domain.

The important feature of the current rewrite is that disjunctions may relate an input state to two potentially distinct output states: s may be related to t via some disjunction in virtue of being related to t via one of its disjuncts. This matters for the meaning of possibility modals (here $\underline{\not\angle}$ denotes any consistent proposition):

$$(\Diamond) \quad \begin{array}{l} s[\Diamond\phi]_i^+ t \text{ iff } t = \{w \in s \colon \langle i, \varnothing \rangle \notin [\phi]_i^+\} \\ s[\Diamond\phi]_i^- t \text{ iff } t = \{w \in s \colon \langle i, \underline{\not\angle} \rangle \notin [\phi]_i^+\} \end{array}$$

As in Update Semantics, the possibility modal is a test. A positive possibility claim requires that the modal domain not be related to the empty state via the prejacent. A negative possibility claim requires that the modal domain only be related to the empty state via the prejacent. Note here that if, for instance, the modal domain i includes no p-worlds, it is related to the empty state via $[p]_i^+$ and thus via $[p \vee q]_i^+$, and so it will violate the test articulated by "$\Diamond(p \vee q)$." This highlights a key innovation over more traditional analyses of possibility modals and disjunction, since we now predict that for a disjunctive possibility to be accepted, both disjuncts must be treated as possibilities, thus accounting for free choice effects and ultimately for why conditionals simplify. The negative entry for the possibility operator accounts for the fact that the free choice effect disappears—disjunction behaves classically—under the scope of negation.

We can then articulate our Ramsey test conditions for conditionals as a combination of a possibility presupposition with an acceptance / rejection claim under the assumption of the antecedent.

$$(>) \quad \begin{array}{l} s[\phi > \psi]_i^+ t \text{ iff } s[\Diamond\phi]_i^+ s \text{ and } t = \{w \in s \colon i \uparrow \phi \Vdash^+ \psi\} \\ s[\phi > \psi]_i^- t \text{ iff } s[\Diamond\phi]_i^+ s \text{ and } t = \{w \in s \colon i \uparrow \phi \Vdash^- \psi\} \end{array}$$

The spirit of this proposal is perfectly familiar, with one noteworthy change: a conditional is now evaluated against the separately provided modal domain i.

It is then easy to state what it takes to update a complex information state:

(i) A complex state π *admits* ϕ, $\pi \rhd \phi$, iff $s_\pi \nVdash^- \phi$

(ii) $u \in \pi + \phi$ iff $u \neq \varnothing$ and $\pi \rhd \phi$ and $\exists s \in \pi \exists i \preceq_\pi s.\, u = \bigcup\{t \colon s[\phi]_i^+ t\}$

(iii) $u \in \pi - \phi$ iff $u \neq \varnothing$ and $\pi \rhd \neg\phi$ and $\exists s \in \pi \exists i \preceq_\pi s.\, u = \bigcup\{t \colon s[\phi]_i^- t\}$

A positive (negative) update of π with ϕ is admitted just in case its center does not reject (the negation of) ϕ. If an update is not admitted, it results in the absurd state; otherwise, we essentially take the result of updating each of its spheres, leaving out the empty set. The only wrinkle: a sphere in π may pass the test imposed by a modal in virtue of a smaller sphere passing the test. This allows the modal domains to evolve dynamically in the way we desire.

Finally, we say what it takes for a complex state to accept and reject a sentence and re-articulate our two notions of validity on that basis.

(iv) $\pi \Vdash^+ \phi$ iff $s_\pi \Vdash^+ \phi$, and $\pi \Vdash^- \phi$ iff $s_\pi \Vdash^- \phi$.

(vi) $\phi_1, \ldots, \phi_n \vDash_1 \psi$ just in case for all π, $\pi + \phi_1 \ldots + \phi_n \vDash^+ \psi$.

(vii) $\phi_1, \ldots, \phi_n \vDash_2 \psi$ just in case for all π, $\pi + \phi_1 \ldots + \phi_n - \psi \vDash^+ \bot$.

Acceptance and rejection by a complex state amount to acceptance and rejection by its center. Note that in defining logical consequence we no longer need to set aside states for which the update fails to be defined in the right way: running an undefined update on a sphere now results in the absurd state.

Everything we said about negated conditionals and conditional negations remains preserved: since $\ulcorner\neg(\phi > \psi)\urcorner$ and $\ulcorner\phi > \neg\psi\urcorner$ share the same presuppositions, they are evaluated against the same minimal sphere and the update conditions once again guarantee that negated conditionals are read as conditional negations and vice versa.

It is straightforward to verify that AS turns out to be invalid in light of these modifications. Consider "$p > r$" and "$(p \wedge q) > r$" and assume that $\pi = \{\{w_1\}, \{w_1, w_2\}\}$ with the following distribution of truth-values:

	p	q	r
w_1	T	F	T
w_2	T	T	F

Clearly $\pi \Vdash^+ p > r$ since the minimal sphere exclusively consists of w_1, which is a $p \wedge r$-world (note that $\{w_1, w_2\}$ remains in π because $\{w_1\}$ is a suitable modal domain). But $\pi + (p \wedge q) > r = \varnothing$ since no sphere in π includes a $p \wedge q$-world and accepts r under the assumption that p and q are true. In fact, $\pi - ((p \wedge q) > r) = \{\{w_1, w_2\}\}$ since $\{w_1, w_2\}$ rejects r under that assumption that p and q are true. (Note furthermore that π admits $(p \wedge q) > r$ and its negation since its center $\{w_1\}$ rejects neither of them due to presupposition failure). Hence AS is neither valid$_1$ nor valid$_2$ and Sobel sequences are consistent.

Furthermore, conditionals with disjunctive antecedents are evaluated against a modal domain that includes both disjuncts as a possibility. For suppose that i does not include any p-worlds: then $i[p]_i^+\varnothing$ and so $i[p \vee q]_i^+\varnothing$ and so i is not a suitable modal domain for a conditional with "$p \vee q$" as its antecedent—and likewise if i does not include any q-worlds. As a consequence, once we have successfully updated with $\ulcorner(\phi \vee \psi) > \chi\urcorner$, the antecedents of $\ulcorner\phi > \chi\urcorner$ and $\ulcorner\psi > \chi\urcorner$ pertain to subsets of the center of π, guaranteeing simplification. IAT remains invalid.

Before I conclude, a few remarks on *might*-conditionals. In the current setting, these cannot be analyzed in terms of the Ramsey conditional and negation, for reasons that are familiar from the Lewis-Stalnaker debate on counterfactuals: as Lewis (1973) observes, if $\phi \diamondto \psi =_{\text{df}} \neg(\phi \boxright \neg\psi)$ and CEM holds, then every *might*-counterfactual $\ulcorner\phi \diamondto \psi\urcorner$ entails its corresponding *would*-counterfactual $\ulcorner\phi \boxright \psi\urcorner$, contrary to the facts.[15] Nonetheless, *might*-conditionals may receive a compositional analysis in the current framework. Specifically, it makes good sense to analyze (19a) as (19b):

(19) a. If it rains, then the match might be cancelled.

[15] I have not said much about the difference between indicative and subjunctive conditionals and I am not going to start now: the obvious proposal is that the latter are evaluated against a system of spheres that is different from but systematically depends on some complex input state π. See, e.g., Willer 2018, Section 4.3 for details.

b. $r > \Diamond c$

In other words, *might*-conditionals are conditionals with a possibility modal in the consequent.

The resulting proposal predicts the incompatibility of $\ulcorner\phi > \psi$ and $\ulcorner\phi > \Diamond\neg\psi\urcorner$. Here again the key observation is that if some state accepts ψ, then updating with the negation of ψ must result in the absurd state, and so will be updating with the claim that the negation of ψ is possible. Likewise, if the possibility of ψ is accepted, then its negation cannot be accepted. This is a good prediction, as Santorio (2017) notes, for instance by pointing to the fact that the following sounds marked:

(20) # If Maria passed, Frida didn't pass; but, even if Maria passed, it might be that Frida passed.

The analysis also predicts that *might*-conditionals simplify in the antecedent and in the consequent:

(21) If Mary or Bill come to the party, it might be fun.
$\rightsquigarrow$ If Mary comes to the party, it might be fun.
$\rightsquigarrow$ If Bill comes to the party, it might be fun.

(22) If Mary does not go to Pisa, she might go to Lisbon or Rome.
$\rightsquigarrow$ If Mary does not go to Pisa, she might go to Lisbon.
$\rightsquigarrow$ If Mary does not go to Pisa, she might go to Rome.

That *might*-conditionals simplify in their antecedents follows from their compositional analysis together with the fact that conditionals with disjunctive antecedents widen the modal horizon so that both disjuncts are live possibilities: hence the simplified conditionals are evaluated against the same modal horizon as their complex cousin. That *might*-conditionals simplify in their consequent follows from their compositional analysis together with the free choice for disjunctive possibilities. On this happy note, let me wrap us the discussion.

5 Conclusion

The framework proposed here—like others before—has taken the Ramsey test as a source inspiration for its story about conditionals, but it adds to the classical Ramseyan question of what it takes to accept a conditional the one of what it takes to reject a conditional. The resulting bilateral semantic analysis of conditionals does not only allow us to disentangle the question of how conditionals play with negation in natural language from the question of whether Conditional Excluded Middle is valid, and without running into Frege-Geach troubles about negation. It also demonstrates that Conditional Excluded Middle is consistent with the spirit of an analysis of conditionals as universal quantifiers over a contextually provided set of possible worlds. Since we can also endorse the intuitive principle of Simplification of Disjunctive Antecedents without negative side effects (which includes avoiding unfortunate antecedent strengthening effects) I conclude that the story told here provides a promising framework for thinking about the conditionals.

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