New Dynamics for Epistemic Modality

Malte Willer

University of Texas, Austin

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Outline

Goal: Provide a Simple and Unifying Account of Disputes about Epistemic Modality and Disputes about Facts

Example (Dispute about Epistemic Modality)

Mary: I can't find my keys.
Alex: They might be in the car.
Mary: No, they can't be in the car. I still had them with me when I came in.

Example (Dispute about Facts)

Mary: I can't find my keys.
Alex: They are in the car.
Mary: No, they are not. I still had them with me when I came in.

Desiderata

- predict that in both cases Mary denies what Alex has asserted
- predict that the dispute arises because Mary and Alex possess different information
- avoid relativism or an *ad hoc* pragmatics for judgements of epistemic modality

A Good Semantics for Epistemic Modals Is Dynamic

Slogan I

Semantics is all about *Context Change Potential* and context change is not always mediated by propositional content.

Slogan II

Epistemically modalised sentences have content, but the content is not truth-conditional content.

This approach:

- avoids the problems of a truth-conditional semantics for epistemic modals
- offers a compositional semantics for epistemic modals
- provides a uniform account of modal and factual disputes
- has some other nice bonus features

Object of Study: Modal Propositional Language

Definition (Language)

$\mathcal{L}$ is the smallest set that contains any sentential atoms $\mathcal{A} = \{p, q, ...\}$ and is closed under negation ($\neg$), conjunction ($\wedge$), and the epistemic modal *might* ($\Diamond$). Disjunction ($\vee$), the material conditional ($\supset$), and the epistemic modal *must* ($\Box$) are defined in the usual way. $\mathcal{L}_0$ is defined as the non-modal fragment of $\mathcal{L}$.

- to take care of all the problematic data, we would need to extend $\mathcal{L}$ with:
  - the natural language conditional
  - some basic tense operators
  - attitude expressions
- but in the interest of time, let's keep things simple

Contexts Are Information States

CCP of *Might*

Might-statements highlight certain epistemic possibilities.

- to learn that ϕ might be the case is to become aware of certain epistemic possibilities
- we say that *might*-statements transform epistemic possibilities into *live* epistemic possibilities
- information states must be fine-grained enough to distinguish between
  - what is merely compatible with what is known
  - live epistemic possibilities
- so information states cannot be *mere* sets of possible worlds

Information States are Sets of Sets of Possible Worlds

Definition (Possible Worlds, States, Information States)

w is a *possible world* iff. $w : \mathcal{A} \mapsto \{0, 1\}$. W is the set of such w's. σ is a *state* iff $\sigma \subseteq W$. S is the set of such σ's. Σ is an *information state* iff $\Sigma \subseteq (S \setminus \emptyset)$. I is the set of such Σ's. The *initial* information state Σ_0 is identical with $(S \setminus \emptyset)$, the *absurd* information state $\Sigma_\emptyset$ with $\emptyset$.

- an information state is a (possibly empty) set of non-empty sets of possible worlds

Possibilities: First Pass

Definition (Possibilities (Special Case))

Consider any $\Sigma \in I$ and $p \in \mathcal{A}$:

1. p is an *epistemic possibility* in Σ iff $\exists w \in \bigcup \Sigma \colon w(p) = 1$
2. p is a *live* epistemic possibility in Σ iff $\exists w \in \bigcup \Sigma \colon w(p) = 1$ and $\forall \sigma \in \Sigma \; \exists w \in \sigma \colon w(p) = 1$

- p is an epistemic possibility if there is at least one set of possible worlds in which p is a possibility
- p is a settled or live epistemic possibility if and only if all sets of possible worlds are such that p is a possibility

A Closer Look

Example ($\mathcal{A} = \{p\}$)

- two possible worlds $w_1(p) = 1$ and $w_2(p) = 0$
- consider the information state $\Sigma = \{\{w_1\}, \{w_2\}, \{w_1, w_2\}\}$
  - p is a possibility
  - p it is not a live epistemic possibility
- learning that p might be the case excludes all those states in which there is no possible world verifying p, i.e. $\{w_2\}$
- learning that p cannot be the case excludes all those states in which there is a possible world verifying p, i.e. $\{w_1\}$, $\{w_1, w_2\}$

- gives us the distinctions we need while preserving a possible worlds model (awesome)

First Step: Updating the Elements of Information States

Definition (Updates on States)

Consider any $\sigma \in S$, $p \in \mathcal{A}$ and $\phi, \psi \in \mathcal{L}$. An update on a state is a function $\uparrow\colon S \mapsto S$ defined by the following recursion:

(1) $\sigma \uparrow p = \{w \in \sigma\colon w(p) = 1\}$
(2) $\sigma \uparrow \neg\phi = \sigma \setminus (\sigma \uparrow \phi)$
(3) $\sigma \uparrow \phi \wedge \psi = (\sigma \uparrow \phi) \uparrow \psi$
(4) $\sigma \uparrow \Diamond\phi = \{w \in \sigma\colon \sigma \uparrow \phi \neq \emptyset\}$

- update with p: eliminate all $w \in \sigma$ sucht $w(p) = 0$
- update with $\ulcorner\neg\phi\urcorner$: take the complement of update with ϕ
- conjunction: functional composition
- update with *might*: running a *test* on a state

Second Step: Updating Information States

Definition (Updates on Information States)

Consider any $\Sigma \in I$ and $\phi \in \mathcal{L}$. An update on an information state is a function $[.] : I \mapsto I$ defined as follows:

$$\Sigma[\phi] = \{\sigma : \sigma \neq \emptyset \wedge \exists \sigma' \in \Sigma : \sigma' \uparrow \phi = \sigma\}$$

- Update of an information state Σ with a formula ϕ thus comes down to the following procedure:
  1. update every element of Σ with ϕ
  2. gather all the resulting non-empty states together

Possibilities: Second Pass

Definition (Possibilities (General Case))

Consider any $\Sigma \in I$ and $\phi \in \mathcal{A}$:

1. ϕ is an *epistemic possibility* in Σ iff $\Sigma[\phi] \neq \emptyset$
2. ϕ is a *live* epistemic possibility in Σ iff $\Sigma[\phi] \neq \emptyset$ and $\forall \sigma \in \Sigma : \sigma \uparrow \phi \neq \emptyset$

- generalises the difference between ϕ being compatible with what is known and ϕ being a live epistemic possibility

Another Example

Example ($\mathcal{A} = \{p\}$)

- $w_1(p) = 1$ and $w_2(p) = 0$
- $\Sigma = \{\{w_1\}, \{w_2\}, \{w_1, w_2\}\}$
  - $\Sigma[p] = \{\{w_1\}\}$
  - $\Sigma[\neg p] = \{\{w_2\}\}$
  - $\Sigma[\Diamond p] = \{\{w_1\}, \{w_1, w_2\}\}$
  - $\Sigma[\neg\Diamond p] = \{\{w_2\}\}$
  - $\Sigma[\Box p] = \{\{w_1\}\}$
  - $\Sigma[\neg\Box p] = \{\{w_2\}, \{w_1, w_2\}\}$

Some Useful Notions

Definition (Settledness, Admission, Entailment)

Let Σ be an information state and ϕ, ψ be formulas:

1. Σ *supports* ϕ, ϕ is *settled* in Σ, $\Sigma \models \phi$, iff $\Sigma[\phi] = \Sigma$
2. Σ *admits* ϕ, $\Sigma \triangleright \phi$, iff $\Sigma \not\models \phi$ and $\Sigma \not\models \neg\phi$
3. ϕ *entails* ψ, $\phi \models \psi$, iff $\forall\Sigma\colon \Sigma[\phi] \models \psi$

- There are three possible relations between a $\Sigma \in I$ and $\phi \in \mathcal{L}$:
  - $\Sigma \models \phi$
  - $\Sigma \triangleright \phi$
  - $\Sigma[\phi] = \emptyset$.

Some Results

Facts

1. For all $\Sigma \neq \Sigma_\emptyset$, $\Sigma \models \Diamond\phi$ iff ϕ is a live epistemic possibility in Σ
2. For all $\phi \in \mathcal{L}_0$: $\phi \models \Box\phi$
3. $\Diamond\phi \not\models \phi$
4. $\Diamond\phi \models \Box\Diamond\phi$

- since $\Sigma[\Diamond\phi] \models \Diamond\phi$, admissible updates with $\ulcorner\Diamond\phi\urcorner$ raise ϕ from an epistemic possibility to a live epistemic possibility
- once $\phi \in \mathcal{L}_0$ is settled, there cannot be any doubt about ϕ
- *might* is non-factive
- the current framework validates the characteristic axiom of S5

The Pragmatics of Assessment Is Very Simple

Assessment

Let $\phi \in \mathcal{L}$ and consider a subject A with information state Σ_A. Then A will by default assess an utterance of ϕ as follows:

- **Agree** in case $\Sigma_A \models \phi$
- **Accept** in case $\Sigma_A \rhd \phi$
- **Reject** in case $\Sigma_A[\phi] = \emptyset$

- A will agree with ϕ if A's information state already encodes the information encoded in ϕ.
- If A's information is incompatible with ϕ, then we should expect that A rejects an assertion of ϕ.
- If A is agnostic about ϕ, then A might as well accept that ϕ is the case.

Disputes about Epistemic Modality Explained

Example (Dispute about Epistemic Modality)

Mary: I can't find my keys.
Alex: They might be in the car.
Mary: No, they can't be in the car. I still had them with me when I came in.

- Mary denies what Alex has asserted
- difference in what is known does not matter for *what* they say when they make their utterances, but for *why* they say it
- $\Sigma_{Alex} \models \Diamond p$, $\Sigma_{Mary} \models \neg p$ and thus $\Sigma_{Mary}[\Diamond p] = \emptyset$
- so no wonder that Mary denies Alex's utterance

Disputes about Facts Explained

Example (Dispute about Facts)

Mary: I can't find my keys.
Alex: They are in the car.
Mary: No, they are not. I still had them with me when I came in.

- Mary denies what Alex has asserted
- difference in what is known does not matter for *what* they say when they make their utterances, but for *why* they say it
- $\Sigma_{Alex} \models p$, $\Sigma_{Mary} \models \neg p$ and thus $\Sigma_{Mary}[p] = \emptyset$
- so no wonder that Mary denies Alex's utterance

Summary

- variation in what is known has everything to do with how Alex's utterance is assessed, and nothing with what has to be the case for the sentence he uttered to be true
- this allows us to give a uniform explanation of disputes about epistemic modality and matters of fact
- epistemically modalised sentences have content, but they aren't true or false
- no general Frege-Geach problem for this non-truth-conditional semantics

More Data I

Example (Ignorance about Epistemic Modality)

Mary: I've heard that John is sick. Might it be cancer?
Alex: I don't know whether it might be cancer. The tests will be in tomorrow.

- John's having cancer is compatible with what Alex and Mary know but...
- John's having cancer is not a live epistemic possibility for Alex and Mary

More Data II

Example (*Might*-Statements Can Be Informative)

Mary: I can't find John. Do you know where he is?
Alex: He might be at home.
Mary: Oh, OK, I call him and check.

- there is a sense in which Alex has provided Mary with some non-trivial information

More Data III

Example (Accepting Judgements of Epistemic Necessity)

Mary: I can't find Colin. Do you know where he is?
Alex: He must in Chicago right now.
Mary: Oh, Ok. What is he doing in Chicago?

- it is compatible with what Mary knows that Colin is not in Chicago
- still, it is natural for Mary not to reject Alex's utterance and instead to uptake the encoded information
- in fact, if Mary has no clue where Colin is, it would be plain weird for her to reject Alex's utterance

More Data IV

Example (Rejecting Judgements of Epistemic Necessity)

Alex: Colin must be at home.
Mary: No, he might be out – maybe he just forgot to turn off the lights.

- Mary rejects Alex's assertion since she thinks (correctly or incorrectly) that he has overlooked a relevant possibility.
- So when is rejection of a judgement of epistemic necessity in order, and when is it not?

The Framework Can Explain All the Data

Definition (Informativity)

Consider any $\phi \in \mathcal{L}$ and $\Sigma \in I$:

ϕ is *informative with respect to* Σ iff $\Sigma[\phi] \neq \Sigma$ and $\Sigma[\phi] \neq \emptyset$

- adding the information encoded in ϕ to Σ should eliminate some but not all elements of Σ

Explaining the Possibility of Ignorance

$\ulcorner\Diamond\phi\urcorner$-admitting Information States

An agent who is in a $\ulcorner\Diamond\phi\urcorner$-admitting information state is agnostic about that sentence – the information the agent possesses neither entails $\ulcorner\Diamond\phi\urcorner$ nor $\ulcorner\neg\Diamond\phi\urcorner$.

- the dialogue about John's condition makes perfect sense
- John's having cancer is compatible with what Alex and Mary know
- but that alone does not make it a live epistemic possibility.
- the possibility of John having cancer is *admitted*, but *not supported* by the relevant information states

Explaining the Informativity of *Might*

Fact ($\ulcorner\Diamond\phi\urcorner$-admitting Information States And Informativity)

For all $\phi \in \mathcal{L}$, $\Sigma \in I$: If $\Sigma \triangleright \phi$, then $\Sigma[\phi] \neq \Sigma$ and $\Sigma[\phi] \neq \emptyset$

- whenever we have a $\ulcorner\Diamond\phi\urcorner$-admitting information state, $\ulcorner\Diamond\phi\urcorner$ will be informative with respect to that information state
- so whenever ϕ is merely compatible with what is known, updating with $\ulcorner\Diamond\phi\urcorner$ will be informative
- the update will highlight certain epistemic possibilities

Explaining *Must* in Discourse I

Fact ($\ulcorner\Diamond\neg\phi\urcorner$-admitting and $\ulcorner\Box\phi\urcorner$-admitting Information States)

For all $\phi \in \mathcal{L}$, $\Sigma \in I$: If $\Sigma \triangleright \Diamond\neg\phi$, then $\Sigma \triangleright \Box\phi$

- Consider a $\ulcorner\Diamond\neg\phi\urcorner$-admitting information state:
  - what is known is compatible with $\ulcorner\neg\phi\urcorner$
  - but the information state does not encode any grounds for believing that $\ulcorner\neg\phi\urcorner$ is a real epistemic possibility
  - thus the agent has no grounds for believing $\ulcorner\neg\Box\phi\urcorner$
- it is merely compatible with Mary's information that Colin isn't in Chicago
- Mary's information admits Alex's claim that Colin must be in Chicago
- we predict that Mary does not – and in fact should not – reject Alex's utterance

Explaining *Must* in Discourse II

Fact ($\ulcorner\Diamond\neg\phi\urcorner$-supporting Information States and $\ulcorner\Box\phi\urcorner$)

For all $\phi \in \mathcal{L}$, $\Sigma \in I$: If $\Sigma \models \Diamond\neg\phi$, then $\Sigma[\Box\phi] = \emptyset$

- Consider a $\ulcorner\Diamond\neg\phi\urcorner$-supporting information state:
  - the agent has grounds for believing $\ulcorner\neg\Box\phi\urcorner$
  - so we expect that an agent who is in such a state will reject a claim that ϕ *must* be the case
- Mary thinks that Colin may have just forgotten to turn off the light, and thus that he might not be at home right now
- so we expect her to reject Alex's assertion that Colin must be at home right now

Some Remaining Work

- We have an elegant and (I think) superior alternatives to current stories about epistemic modality, but it is incomplete.
- How agents assess previously made judgements of epistemic modality in the light of new information is quite complicated.
  - add some basic tense operators to the framework
- Epistemic modals may occur in conditionals and under the scope of attitude ascriptions.
  - provide a dynamic semantics for conditionals
  - explain what it is to believe that ϕ might/must be the case

Highlights

- non-truth-conditional semantics for epistemic modality is in great shape
- explains disagreement about epistemic modality without relativism or an overly weak pragmatics
- no general embedding problem
- other goodies too