

# An Update on Epistemic Modals\*

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## Abstract

Epistemic modals are a prominent topic in the literature on natural language semantics, with wide-ranging implications for issues in philosophy of language and philosophical logic. Considerations about the role that epistemic *might* and *must* play in discourse and reasoning have led to the development of several important alternatives to classical possible worlds semantics for natural language modal expressions. This is an opinionated overview of what I take to be some of the most exciting issues and developments in the field.

## 1 Introduction

Epistemic modals—modals that express what, in light of some body of information, *might* or *must* be the case—are a prominent topic in the literature on natural language semantics, with wide-ranging implications for issues in philosophy of language and philosophical logic. Considerations about the role that epistemic *might* and *must* play in discourse and reasoning have led to the development of several important alternatives to classical possible worlds semantics for natural language modal expressions. This classical approach, going back to the seminal work by Kratzer, treats epistemic modals as existential or universal quantifiers over a set of possible worlds projected from a contextually salient body of evidence. So if  $c$  is a context of utterance and  $w \in W$  is a world of evaluation, the interpretation function for epistemic *might* looks as follows (as usual, treat *might* and *must* as duals):

$$\llbracket \Diamond \phi \rrbracket^{c,w} = 1 \text{ iff } \exists v: wRv \ \& \ v \in \llbracket \phi \rrbracket^c$$

where  $wRv$  just in case  $v$  is compatible with the  $c$ -relevant knowledge at  $w$ . This view is labeled ‘contextualism’ for obvious reasons, and it is common to distinguish between *solipsistic* and *nonsolipsistic* versions of this view. According to the former, the  $c$ -relevant knowledge is only the speaker’s; according to the latter, it is the knowledge distributed among a contextually salient group. What is common to these views is that the content of a judgment of epistemic modality is fixed by the context in which it is made.<sup>1</sup>

A good deal of the literature on epistemic modals addresses the question whether a contextualist approach to epistemic modals is tenable. The most prominent challenge is

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<sup>1</sup>Versions of contextualism are developed in [Hacking 1967](#), [Teller 1972](#), [DeRose 1991](#), and [Schaffer 2011](#). Kratzer’s analysis further restricts the domain of quantification to the *closest* worlds in the modal base, to be determined by a contextually provided ordering source, but this matter of detail need not detain us here. See [Kratzer 2012](#) for an up-to-date discussion of her views.

the *problem of modal disagreement* (see Egan et al. 2005, Egan 2007, Stephenson 2008, and MacFarlane 2011, 2014). Consider the following discourse:

- (1) Alice: I can't find Charles. Do you know where he is?  
 Bob: He might be in Boston.  
 Alice: No, he can't be in Boston. I just saw him this morning.

Contextualism has a hard time accounting for the intuition that Alice denies what Bob has asserted, namely that Charles might be in Boston. On its solipsistic version there is no disagreement since what Alice and Bob say is perfectly compatible if all they do is report on what is or is not ruled out by what they respectively know. Nonsolipsistic contextualism can account for the intuition that there is real disagreement, but only if Bob is interpreted as reporting on Alice's knowledge. And that does not seem right since Bob produces his judgment of epistemic modality on the basis of his own information state and has no business reporting on what Alice knows. The dilemma: there seems to be no single contextual parameter that explains how a judgment of epistemic modality is produced and how it is assessed in everyday discourse.

Some have concluded that the disagreement puzzle favors *relativism* over contextualism. Relativists assign to epistemic modals truth-values relative to a context of utterance, world of evaluation, and point of assessment (a judge or some other parameter that varies with the assessor):

$$\llbracket \Diamond \phi \rrbracket^{c,w,a} = 1 \text{ iff } \exists v: wRv \ \& \ v \in \llbracket \phi \rrbracket^{c,a}$$

where  $wRv$  just in case  $v$  is compatible with what  $j_a$  knows at  $w$ . Relativism allows for the truth-value of an epistemic modal claim to vary in truth-value across a parameter that floats free of the context of utterance and world of evaluation. It thereby avoids the problem contextualism faces—that there is no single contextual parameter explaining how judgments of epistemic modality are produced and how they are assessed in discourse—in a rather obvious manner. Bob's claim that Charles might be in Boston is literally true when he is the assessor and literally false when Alice is (and vice versa for Alice's utterance). The disagreement between Alice and Bob is real since there is a single proposition on which they are disputing, albeit one that varies in truth-value across points of assessments.<sup>2</sup>

Even if one takes the problem of modal disagreement seriously, there is reason to think that the move towards relativism is a bit of an overreaction and that the data can be accommodated in a semantically more conservative framework. For instance, von Fintel and Gillies (2011) challenge the assumption that a single contextual parameter must account for how a judgment of epistemic modality is produced and how it is assessed in discourse—indeed, it is hard to see what this parameter should be in the first place since the facts about a conversation constrain but often *underdetermine* whose knowledge is contextually relevant for the truth-value of an epistemic modal claim. On von Fintel and Gillies's view, utterances that are contextually underspecified 'put into play' the propositions corresponding to the various disambiguations. For a speaker to be in a position to *assert* an epistemic modal, at least one of the propositions put into play must

<sup>2</sup>See again Egan et al. 2005, Egan 2007, Stephenson 2008, and MacFarlane 2011, 2014. I set aside here versions of relativism that allow for variation of content rather than of truth-value across points of assessment. See MacFarlane (2009) for discussion.

be true. A hearer can *confirm* (*deny*) the assertion in case the strongest proposition that is put into play and that the hearer reasonably has an opinion about is such that the hearer thinks it is *true* (*false*). In (1), for instance, Bob’s assertion is justified in virtue of its true solipsistic contextualist disambiguation, but at the same time its false nonsolipsistic contextualist disambiguation—Alice, after all, knows that Charles is not in Boston—justifies his interlocutor’s negative assessment. The crucial idea here is that what licenses an assertion of *might* is a sharpening of an underspecified contextual parameter that need not be identical to the one licensing the denial of that very same speech act.<sup>3</sup>

One important lesson, then, is that the puzzle of modal disagreement leaves quite some room for contextualist manoeuvre. The problem is nonetheless instructive since it highlights a surprisingly simple issue that arises once we combine the observation that speakers reliably produce and assess epistemic modal claims on the basis of their own states of information with the assumption that these claims are *descriptions* of such states. Instead of adopting relativism or embellishing contextualism with a very flexible pragmatics to allow for meaningful discussion about what might or must be the case, one may thus challenge the underlying descriptive interpretation of epistemic modal claims (see Willer 2013). This strategy gains some additional support from the *problem of epistemic contradictions*, which affects contextualism and relativism alike. Yalcin (2007, 2011) observes that (2a) sounds terrible when asserted and also when supposed:

- (2) a. # It is raining and it might not be raining.  
 b. # Suppose it is raining and it might not be raining.

The fact that (2a) cannot even be supposed blocks a classical pragmatic explanation of its infelicity that is familiar from discussions of Moore’s paradox, and so it seems to be the job of our semantics for *might* and *must* to tell us why (2a) is marked. But this is not easy to do if we think of epistemic modals as descriptions of what is or is not compatible with some state of information, since there is nothing strange about supposing that there are unknown truths.

Given the problems that a descriptivist semantics for epistemic modals needs to deal with, it is then not surprising that the literature contains a healthy number of sophisticated non-descriptive alternatives. Despite their differences, most of these frameworks owe a lot of inspiration to the seminal *dynamic* analysis of *might* and *must* by Veltman (1996), which I will outline in the next section. At a later stage, we will consider how the dynamic proposal relates to accounts that assign to epistemic modals truth-conditions relative to a separate *nonfactual* informational parameter and offer an *expressivist* interpretation of epistemic modal claims.

## 2 Update Semantics

A common way of motivating a dynamic perspective on meaning and communication starts with Stalnaker’s (1978) observation that context-content interaction is not a one-way road: context determines which proposition an assertion expresses, but assertions in turn affect the context, and they do so by adding the proposition expressed to the common ground (what is mutually presupposed). On Stalnaker’s view, the effect that

<sup>3</sup>See also Wright 2007, Dietz 2008, von Stechow 2008a, and Dowell 2011 for critical discussion of the problem of modal disagreement and whether it promotes relativism.

assertions have on the conversational score is a pragmatic affair and always mediated by propositional content, but his picture about context-content interaction also inspires a change of perspective: instead of modeling what is said in terms of truth-conditions, a semantics may capture the meaning of a sentence in terms of its effects on contexts. Meanings then become relational: they are relations between an input to an output state. This is just what happens in Veltman’s update semantics.

Veltman defines an update function for a simple modal propositional language  $\mathcal{L}$  whose modal operators are interpreted epistemically. Input and output states are sets of possible worlds. Each  $\phi \in \mathcal{L}$  is then associated with a state change rule  $[\phi]: \mathcal{P}(W) \mapsto \mathcal{P}(W)$ :

1.  $i[p] = \{w \in i: w(p) = 1\}$
2.  $i[\neg\phi] = i \setminus i[\phi]$
3.  $i[\phi \wedge \psi] = i[\phi][\psi]$ <sup>4</sup>
4.  $i[\diamond\phi] = \{w \in i: i[\phi] \neq \emptyset\}$

Notice here that an update with *might* imposes a test on the input state. If the state is compatible with the prejacent, the test is passed; otherwise we get back the empty set.

A crucial observation is that  $[\cdot]$  as defined is eliminative but not distributive:

5.  $[\cdot]$  is *eliminative* iff for any state  $i$  and any sentence  $\phi$ :  $i[\phi] \subseteq i$
6.  $[\cdot]$  is *distributive* iff for any state  $i$  and any sentence  $\phi$ :  $i[\phi] = \bigcup_{w \in i} \{w\}[\phi]$

Distributivity failures result from the update rule for *might*. Consider  $i = \{w_1, w_2\}$  so that  $w_1(p) = 1$  and  $w_2(p) = 0$ : then  $i[\diamond p] = i$  but  $\bigcup_{w \in i} \{w\}[\diamond p] = \{w_1\}$ . A well known-result from [van Benthem \(1986\)](#) shows that eliminativity and distributivity are necessary and jointly sufficient conditions for  $[\cdot]$  being static in the following sense (see also [Groenendijk and Stokhof 1991](#), [von Fintel and Gillies 2008b](#), and [Muskens et al. 2011](#)):

7.  $[\cdot]$  is *static* iff for any state  $i$  and any sentence  $\phi$ :  $i[\phi] = i \cap W[\phi]$

If  $[\cdot]$  is static, update with  $\phi$  amounts to adding a truth-condition determining proposition  $\llbracket\phi\rrbracket$  to the input state of information. In that case, the shift from classical truth-conditional to dynamic semantics fails to offer a genuine shift of perspective: the primary task of a semantic theory remains to associate with each sentence  $\phi$  a set of possible worlds at which it is true. At the same time, the previous observation about distributivity failures in update semantics shows that Veltman’s account—and in particular its analysis of the epistemic modals *might* and *must*—constitutes a departure from classical truth-conditional semantics.

If meaning is an update operation on states that fails to be static, one has to depart from the classical conception of validity as necessary preservation of truth at a point of evaluation. Veltman offers various alternatives (see also [van Benthem 1996](#)) but two options are of particular interest:

8.  $i \models \phi$ ,  $\phi$  is *accepted* in  $i$ , iff  $i[\phi] = i$

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<sup>4</sup>Veltman’s original update rule for conjunction differs in that it treats conjunction as internally static: updating  $i$  with a  $\ulcorner\phi \wedge \psi\urcorner$  amounts to taking the intersection of  $i[\phi]$  and  $i[\psi]$ . This difference need not detain us here.

9.  $\phi_1, \dots, \phi_n \models_{\text{ttc}} \psi$  iff  $\forall i$ : if  $i \models \phi_1$  and ... and  $i \models \phi_n$ , then  $i \models \psi$   
 10.  $\phi_1, \dots, \phi_n \models_{\text{utc}} \psi$  iff  $\forall i$ :  $i[\phi_1] \dots [\phi_n] \models \psi$

Say that  $\phi$  is accepted in  $i$  just in case the information carried by  $\phi$  is already carried by  $i$ . We may then say that an argument is valid just in case every state in which its premises are accepted also accepts its conclusion (*test-to-test consequence*). Alternatively, we may say that an argument is valid just in case the result of updating any information state with its premises accepts the conclusion (*update-to-test consequence*). Does et al. (1997) develop sequent-style proof systems for both notions of logical consequence that are not only sound and complete, but also decidable (see also van Benthem 1989 and van Eijk and de Vries 1995 for important discussions) .

In light of the difference between test-to-test and update-to-test consequence, we can distinguish between coherent and consistent sequences:

11.  $\phi_1, \dots, \phi_n$  is *coherent* iff  $\phi_1, \dots, \phi_n \not\models_{\text{ttc}} \perp$   
 12.  $\phi_1, \dots, \phi_n$  is *consistent* iff  $\phi_1, \dots, \phi_n \not\models_{\text{utc}} \perp$

Coherence entails consistency but not the other way round. Update semantics then predicts that there is a contrast between (3) and (4):

- (3) It might be raining. ... It is not raining.  
 (4) It is not raining. ... # It might be raining.

While (3) is consistent, (4) is not. This is just what we have in update semantics: updating any  $i$  with  $\neg p$  and then with  $\Diamond p$  results in the absurd state (the empty set); updating  $i$  with  $\Diamond p$  and then with  $\neg p$  is not guaranteed to have the same result. Notice, however, that the result of updating  $i$  with the sequence in (3) does not support “It might be raining” and so (3) is incoherent. It follows straightaway that update-to-test consequence is nonmonotonic. The underlying fact is that epistemic *might* fails to be *persistent*:

A sentence  $\phi$  is persistent with respect to  $\models$  iff for all states  $i$ : if  $i \models \phi$  and  $i' \subseteq i$ , then  $i' \models \phi$ .

If states are interpreted as models of belief, change in belief is not doxastically conservative: doxastic commitments may be defeated even if the additional bit of information is consistent with what is believed. Gillies (2006) develops a theory of belief change that is inspired by update semantics and elegantly avoids the problem that classical AGM belief revision models have with introspective agents (see Levi 1988, Fuhrmann 1989, Rott 1989).

Veltman’s update semantics has a surprisingly easy time avoiding the problems that classical contextualism faces. Notice that it follows straightaway from the definition of negation as set subtraction that that  $\Diamond p$  contradicts  $\neg \Diamond p$ . Alice denies what Bob has asserted, and we can make sense of this since they are not interpreted as reporting (truly or falsely) on someone’s knowledge, but rather as imposing incompatible constraints on a common conversational score. And of course, both Alice’s and Bob’s conversational moves are perfectly justified in light of their individual knowledge: while her state of information supports  $\neg p$  and thus  $\neg \Diamond p$ , his state of information supports  $\Diamond p$ .

It is just as straightforward to verify that, given dynamic conjunction, a sentence like “It is raining and it might not be raining” is predicted to be a straight contradiction in update semantics. Notice, however, that the resulting proposal also predicts that reversing the order of conjuncts in Yalcin’s example is consistent:

(5) # It might not be raining and it is raining.

It is unclear whether there is a robust difference between (5) and Yalcin’s original example. Reversing the order of conjuncts sometimes improves the odd sounding (2a) (see [Sorensen \(2009\)](#) and [Dorr and Hawthorne \(2013\)](#) for discussion), but (5) still strikes many speakers as odd to assert. The good news is that both sentences are incoherent and thus, insofar as the point of a supposition is to adopt a state of information that supports a certain hypothesis, odd to assume. And on the plausible assumption—inspired by the discussion in [Shoemaker 1995](#)—that one can only coherently assert what one can also coherently suppose, it also follows that plain assertions of these sentences are equally infelicitous.

It is hard to overstate the impact of Veltman’s treatment of *might* on subsequent work addressing issues surrounding epistemic modality. [Groenendijk et al. \(1996\)](#) integrate dynamic predicate logic into the framework of update semantics; [Zeevat \(1992\)](#) and [Beaver \(2001\)](#) expand Veltman’s account to cover presuppositions; Gillies uses update semantics in his treatment of Moore’s paradox (2001) and indicative conditionals (2004). A number of papers also try to overcome an important limitation of Veltman’s account: epistemic *might* fails to induce non-trivial changes on the input state, in the sense that the update result is either the original set or the absurd state (similar worries apply to epistemic *must*).<sup>5</sup> It thus remains unclear how *might* can be used to make informative moves in discourse. [Yablo \(2011\)](#) allows for epistemic *might* to expand the input state, but he does not say how negation is supposed to interact with the resulting non-eliminative update rule for the epistemic possibility operator. Alternatively, one may refine the notion of an information state on which updates operate. [Yalcin \(2011\)](#) adds partitions to an information state that keep track of what propositions are ‘visible’ in discourse and reasoning. The update rule for *might* could then be refined so that it is not only a test but also raises issues. [Willer \(2013\)](#) models input states as sets of sets of possible worlds and defines a difference between live (or explicitly recognized) possibilities and those that are merely compatible with what is presupposed in a supervaluationist fashion. So if  $I$  is an input state understood as a set of sets of possible worlds and  $\phi$  an element of the propositional fragment of  $\mathcal{L}$ , then:

13.  $\phi$  is compatible with what is presupposed in  $I$  iff  $\exists i \in I \exists w \in i: w \in \llbracket \phi \rrbracket$
14.  $\phi$  is a live (or explicitly recognized) possibility in  $I$  iff  $\forall i \in I \exists w \in i: w \in \llbracket \phi \rrbracket$

Such input states are then updated by updating each of their members in the familiar fashion:

$$I \uparrow \phi = \{i: i \neq \emptyset \wedge \exists i' \in I: i'[\phi] = i\}$$

On this view, epistemic *might* is designed to change its prejacent from a possibility that is merely compatible with what is presupposed to a live possibility.

<sup>5</sup>Update semantics also predicts that embedding an epistemic modal under another epistemic modal has no interesting semantic effects. A careful analysis of this prediction must be left to another day but see [Moss 2015](#) for critical discussion.

### 3 Information-Sensitive Truth-Conditions

Veltman’s framework models semantic values as state change rules, that is, properties of information states. Whenever an update with  $\phi$  is static,  $[\phi]$  is a property of information states that is also a property of possible worlds, but this is not so in case updating with  $\phi$  fails to be static (cf. van Benthem’s (1986) discussion of adjectival phrases). Notice that, because of eliminativity,  $i[\phi] = i \cap i[\phi]$  and so  $i[\phi] = i \cap \{w : w \in i[\phi]\}$ . Instead of assigning to each  $\phi \in \mathcal{L}$  a dynamic state change rule, we may then assign a static semantic value—a set of possible worlds—relative to an informational parameter  $i$  as follows:  $w \in \llbracket \phi \rrbracket^i$  iff  $w \in i[\phi]$ . The resulting truth-conditions for  $\mathcal{L}$  predict that  $\phi$  false whenever  $w \notin i$ , but it is unproblematic to restrict the domain of  $\llbracket \phi \rrbracket$  to world-information state pairs  $\langle w, i \rangle$  that are ‘proper’ in the sense that  $w \in i$ . If we now define a state change rule  $|\cdot|$  modeling update of  $i$  with  $\phi$  as intersecting  $i$  with  $\llbracket \phi \rrbracket^i$ :

$$i|\phi| = i \cap \llbracket \phi \rrbracket^i$$

it follows immediately that for every state  $i$  and sentence  $\phi$ ,  $i[\phi] = i|\phi|$ . Notice here that  $|\cdot|$  fails to be distributive since for some  $i$  and  $\phi$ ,  $i \cap \llbracket \phi \rrbracket^i \neq \bigcup_{w \in i} \{w\} \cap \llbracket \phi \rrbracket^{\{w\}}$ .

The preceding observation suggests that there is an interesting connection between update semantics and the recent trend in the literature of assigning to epistemic modals truth-values relative to a separate informational parameter. One can see this more clearly by considering the proposal by Yalcin (2007, 2011), who treats epistemic modals as specifications of informational modals, that is, quantifiers whose domain is provided by a separate informational parameter. The semantics for epistemic *might* is as follows:

$$\llbracket \diamond \phi \rrbracket^{c,w,i} = 1 \text{ iff } \exists v \in i : \llbracket \phi \rrbracket^{c,v,i} = 1$$

Observe that the truth-values of epistemic modal operators are sensitive to a parameter that floats free of the relevant contextual parameter, and in fact it is easy to see that if we set context-sensitivity aside and only consider proper points of evaluation, Yalcin’s truth-conditions for epistemic modals are equivalent to those we derived from update semantics.

The resulting proposal looks suspiciously close to the relativist one we considered in §1, and so one might worry that everything we said so far just leads back to descriptivism about epistemic modals. But this worry is groundless, since there is in fact a substantial difference between Yalcin’s account (and, correspondingly, the static truth-conditions we derived from update semantics) and relativism. For the latter, the facts about a conversation provide a point of assessment for an epistemic modal claim, albeit one that may differ between producer and assessor, and epistemic modal claims express propositions, even though they may vary in truth-value across points of assessments. In contrast, Yalcin treats the informational parameter as *nonfactual* in the sense that it does not correspond to any possible feature of the context of utterance or point of assessment: the facts about a conversation neither fix nor constrain it. As a consequence, epistemic modal claims do not describe what is or is not compatible with some distinct state of information, that is, they do not articulate propositions.<sup>6</sup> But what *do* such claims communicate, then? Yalcin

<sup>6</sup>For parallel reasons, the proposal presently under consideration differs from a contextualist account of epistemic modals, which in principle may accept the abstract semantic framework but would then think of the informational parameter as fixed (or at least constrained) by the context of utterance.

offers an *expressivist* interpretation: they express states of mind and are made with the objective of coordinating the common ground on a property of such states—in the case of epistemic *might*, the property of leaving a certain possibility open. Generalizing a bit, we may say that all utterances of a sentence  $\phi$  in context  $c$  express a state of accepting  $\phi_c$ :

$$\phi_c \text{ is accepted in } i \text{ iff } \forall w \in i: \llbracket \phi \rrbracket^{c,w,i} = 1$$

Notice here that  $\phi_c$  is accepted in  $i$  just in case  $i \cap \llbracket \phi \rrbracket^{c,i} = i$ . The current notion of acceptance is thus equivalent to the one in update semantics, where  $i \models \phi$  just in case  $i[\phi] = i$ , which given our earlier results is just to require that  $i|\phi| = i$  and so that  $i \cap \llbracket \phi \rrbracket^i = i$  (again modulo context-sensitivity). This is important since it shows that, as in Veltman’s framework, accepting a *might*-claim is here a *global* property of one’s state of mind that does not reduce to believing some proposition expressed by that claim. Specifically, believing that something might be the case—being in a state of mind that leaves a certain possibility open—is not to have some belief *inter alia* about one’s own state of mind, as it is on the descriptive accounts considered in §1. Accordingly, epistemic *might*-claims are not efforts to add some proposition to the conversational score, but rather aim at coordination on a non-local feature of the common ground.

While the expressivist proposal considered here differs in detail and emphasis from the dynamic approach to epistemic modals, there is reason to think that the two accounts can complement each other. In fact, a dynamic compositional semantics for epistemic modals goes naturally together with the proposal that *might*-claims express certain states of mind and are made with the aim of coordinating the common ground on a certain non-local property of such states (that is, the property of leaving some distinct possibility open). An expressivist account, in turn, is compatible with the idea that the states expressed by an epistemic modal claim are individuated by dynamic state change rules. Those rules would then immediately identify the properties of the common ground that the epistemic modal claim aims to coordinate on. All of this suggests that the expressivist and the dynamic agenda, while commonly pursued in isolation by researchers with differing theoretical goals and interests, have a lot to learn from each other.

## 4 Further Issues

So far I have focussed on foundational questions about the semantics and pragmatics of epistemic modals. Here I will briefly mention a few other prominent issues.

### 4.1 Must and Evidentiality

In the frameworks developed by Veltman (1996) and Yalcin (2007, 2011) (who adopts a version of test-to-test consequence),  $p$  entails  $\Box p$  and vice versa. The left-to-right direction receives support from the role it plays in accounting for the inconsistency of the sequence (3). The right-to-left direction, however, conflicts with the often voiced impression that a claim like “It must be raining” seems to say something weaker than the unmodalized “It is raining” (see Karttunen 1972, among others). This impression is accounted for by the semantic analysis of epistemic necessity by Kratzer and also by Veltman in earlier work (see Veltman 1985). However, the result that  $\Box p$  entails  $p$  has been defended by von

Fintel and Gillies (2010), who also address the observation that epistemic *must* carries an evidential signal. The case they consider starts with the observation that if Alice is looking out of the window seeing pouring rain, she can assert (6a) but not (6b):

- (6) a. It is raining.  
 b. It must be raining.

If instead she sees people coming from outside with wet raingear, both (6a) and (6b) are acceptable, even if it is clear that rain is the only possible explanation.

The proposal endorsed by von Fintel and Gillies treats epistemic modals as presupposing that the prejacent is not settled by the direct evidence. This explains why (6b) is not assertible in case the speaker sees that it is raining, but assertible in case the speaker infers that it is raining. However, and as von Fintel and Gillies recognize, treating the evidential component of epistemic modals as a presupposition and thus as part of the lexical semantics has a hard time accounting for the observation that epistemic modals signal indirect inference across distinct languages. But it is not obvious what other formal status the evidential marking of indirect inference could have.

There is an extensive literature on evidentials, which cannot be adequately addressed here. Let me briefly mention, however, that there is an ongoing debate about the relation between evidentiality and epistemic modality (see Speas 2008 for an overview). For instance, while some researchers analyze evidentials as illocutionary force operators that do not contribute to core meaning (see, e.g., Faller 2002 and Davis et al. 2007), others have suggested that at least some evidentials are in fact epistemic modals with an extra meaning component (McCready and Ogata 2007 and Matthewson et al. 2008, among others). Needless to say, the fact that there is no consensus on how epistemic modals are to be semantically analyzed in the first place does not make it easier to resolve this debate.

## 4.2 Embedding Epistemic Modals

Epistemic modals embed under other operators, though not as freely as other modals (see Hacquard and Wellwood 2012 for a corpus study and an overview of the literature on embeddings of epistemic modals). For instance, epistemic modals may occur in questions, in the antecedent of conditionals, under the scope of attitudes, and under the scope of quantifiers:

- (7) a. Must Alfred have cancer? (Papafragou 2006)  
 b. If there might have been a mistake, the editor will have to reread the manuscript. (von Fintel and Gillies 2008b)  
 c. Sam thinks that it might be raining. (Stephenson 2007)  
 d. Be careful where you step, because every inch of the floor might have paint on it. (Swanson 2011)

Notice here that (7d) is assertible if paint could be anywhere but one is certain that paint has not been splattered everywhere. In that case, the only available reading is one on which *might* scopes under the quantifier.

Epistemic modals may also scope under negation and temporal operators. While English *might* and *must* resist embedding under negation, *have to* does not exhibit this

restriction even if interpreted epistemically, and neither do German *können* and *müssen*. [Condoravdi \(2002\)](#) observes that a modal like *might have* allows for multiple interpretations due to scope ambiguity but maintains that *might* receives an epistemic interpretation only in case it takes wide scope; if it scopes under *have*, a metaphysical reading is mandatory. This is in line with the majority view, going back at least to the discussion by [Groenendijk and Stokhof \(1975\)](#), that epistemic modals cannot scope under past tense. However, such cases are attested across languages. To give an example from [von Fintel and Gillies 2008a](#), consider a case in which Sophie checks the freezer for ice cream but does not find any. If asked why she opened the freezer, Sophie may reply as follows:

- (8) There might have been ice cream in the freezer.

What Sophie is describing here is not an epistemically possible past—she now knows that there was no ice cream in the freezer, and so do we—but a past epistemic possibility.

The embedding facts put pressure on proposals to treat epistemic modals as ‘comments’ that do not contribute to semantic content but rather indicate the degree or source of the speaker’s commitment to the embedded proposition (for such ‘force modifier’ analyses, see [Price 1983](#), [Westmoreland 1998](#), [Drubig 2001](#), and [Huitink 2008](#), but also [Kant 1781](#) and [Frege 1879](#)). The previous remarks about update semantics should make it clear, however, that embedding facts cannot serve as a silver bullet against all non-truth-conditional analyses of epistemic modals. In fact, there is reason to think that certain embedding phenomena receive an attractive treatment in dynamic semantics. Observe that (9a) together with (9b) does not entail (9c) (as noted by [Klinedinst and Rothschild \(2012\)](#), who credit [Yalcin](#)):

- (9) a. Either Mary is in Chicago or she must be in New York.  
 b. Mary might not be in New York.  
 c. Mary is in Chicago.

Assume the familiar definition of disjunction in terms of negation and conjunction. Then the inference of (9c) from (9a) and (9b) is classically valid unless we can make a case that *must* scopes over the disjunction in (9a). In contrast, if conjunction is interpreted dynamically, the update rule for disjunction turns out to be as follows:

$$i[\phi \vee \psi] = i[\phi] \cup i[\neg\phi][\psi]$$

It follows straightaway that modus tollendo ponens fails and so that (9c) does not follow from (9a) and (9b) even if taken on their obvious logical form.

The preceding considerations naturally motivate a corresponding dynamic analysis of conjunction and disjunction in truth-conditional semantics. If this is right, we have independent evidence that a truth-conditional analysis of epistemic modals must invoke a parameter that evolves dynamically in discourse and reasoning (see [Klinedinst and Rothschild 2012](#) for such an analysis).

### 4.3 The Scope of Epistemic Modality

Several authors have observed that the language of subjective uncertainty includes terms such as *likely*, *certain*, and *probably* (see [Swanson 2011](#) in addition to the sources cited below). A distinguishing feature of these expressions that receives a lot of attention in the

current literature is their gradability. For instance, an event may not only be certain, but very/somehow/less than 50% certain. While Kratzer’s classical analysis of modality allows for ordering sources to impose a comparative order on the space of possibilities, several authors have argued that this framework is inadequate for gradable epistemic modals. The most prominent alternative is to interpret such modals on the basis of probability theory. Yalcin enriches information states with a probability structure that figures in the interpretation of *probably* but essentially preserves a quantificational analysis of *might* and *must* (see the works already cited and also Yalcin 2010, 2012). Lassiter (2011) argues that *all* epistemic modals denote measure functions from propositions to the probability scale  $[0, 1]$ , thus advocating for a semantic analysis of epistemic modals that differs radically not only from Kratzer’s but also from the other proposals considered so far. Klecha (2012) argues that *certain* and *likely*, while gradable, need to be interpreted with respect to different scales, and he advocates for a semantics that is flexible enough to cover gradable as well as non-gradable modals (see also Portner 2009 for relevant discussion). It is to be expected that the outcome of this discussion will substantially influence how philosophers, linguists, and logicians think about the language of subjective uncertainty more generally.

Issues pertaining to epistemic modality are also prominent in recent discussions about deontic modals. This development is foreshadowed by Thomason (1981a,b), who observes that deontic *ought* allows for a deliberative interpretation—the question is what ought to be done taking the facts as given—and a non-deliberative interpretation on which the facts themselves are subject to evaluation. Deliberative interpretations of deontic *ought* figure in a recent paradox about deontic conditionals from Kolodny and MacFarlane (2010). Ten miners are trapped either in shaft *A* or in shaft *B*, but we do not know which one. Water threatens to flood the shafts. We only have enough sandbags to block one shaft but not both. If one shaft is blocked, all of the water will go into the other shaft, killing every miner inside. If we block neither shaft, both will be partially flooded, killing one miner. In this scenario, all of the following sound true:

- (10) a. We ought to block neither shaft.
- b. If the miners are in shaft *A*, we ought to block shaft *A*.
- c. If the miners are in shaft *B*, we ought to block shaft *B*.

However, from a classical perspective this is an inconsistent triad. Dilemma reasoning allows us to derive that we ought to block shaft *A* or ought to block shaft *B*, which contradicts (10a) given the D-axiom of deontic logic.

Kolodny and MacFarlane analyze deontic modals as specifications of informational modals that are alike to epistemic modals on Yalcin’s account in that their quantificational domain is determined by a separate informational parameter. On this conception, what is deontically ideal is highly sensitive to what is taken for granted in discourse and reasoning, and several semantic frameworks capturing this idea have been developed (see Cariani et al. 2013, Charlow 2013, and Silk 2014; for critical discussion, see Dowell 2012 and von Fintel 2012). If this is right, it is to be expected that the extensive research on epistemic modality fruitfully interacts with questions about deontic modality. For instance, while Kolodny and MacFarlane treat the miners paradox as a conclusive counterexample to modus ponens, Willer (2012) shows that a dynamic conception of logical consequence inspired by Veltman’s update-to-test consequence preserves the validity of modus ponens but avoids the miners paradox by blocking dilemma reasoning.

## 5 Conclusion

A key hypothesis that I have pursued here at some length is that epistemic modal claims are not uttered with the intention of describing some state of information but are first and foremost tools for coordinating on a feature of such states that does not reduce to a property of classical points of evaluation. This hypothesis motivates a major departure from a classical truth-conditional semantics for epistemic modals. But we have seen that there are several other challenges that a semantics and pragmatics for epistemic modals needs to address. It needs to explain how such modals compositionally interact with other operators, and there is some evidence that this interaction is dynamic in nature and effectively favors a non-classical logic that is sensitive to the flow of information in discourse and reasoning. Epistemic modality is closely intertwined with evidentiality and gradability phenomena in natural language, though the exact nature of this relation is the subject of an ongoing debate. On the other hand, the progress that has already been made in understanding the nature of epistemic modality continues to have a genuine impact in philosophy, linguistics, and logic. Given the current momentum of the field, it is to be expected that the present update on epistemic modals is bound to be outdated in the not too distant future.

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