Dynamic Thoughts on Ifs and Oughts

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Abstract

Iffy oughts figure prominently in a variety of paradoxes of deontic logic. A satisfying response to these paradoxes is a non-negotiable component of any adequate semantic story about conditionals and deontic modals. I demonstrate that such a story can be told but doing so requires that we supplement a semantics that pays proper attention to the sensitivity of if and oughts to contextual information with a dynamic conception of logical consequence. The resulting framework naturally leads to a nonmonotonic logic for conditionals and deontic modals which differs from its static alternatives in that it elegantly resolves not only Kolodny’s and MacFarlane’s recent miners paradox but also the more classical paradoxes about conditional obligations from Forrester and Chisholm.

1 The Plot

Iffy oughts figure prominently in a variety of paradoxes of deontic logic: Chisholm’s (1963) Contrary-to-Duty Paradox, Forrester’s (1984) Gentle Murder Paradox, as well as the recent Miners Paradox from Kolodny and MacFarlane (2010). A satisfying response to these paradoxes is a non-negotiable component of any adequate semantic story about conditionals and deontic modals. I will demonstrate that such a story can be told and along the way explain a bit why deontic conditionals are prone to so much trouble. Very roughly, conditionals and deontic modals are sensitive to the information that is taken for granted in discourse and reasoning and as such the right semantics for if and oughts must pay proper attention to the complex mutual interaction between context and content—something that classical semantic frameworks for modality fail to do. A thoroughly dynamic perspective on discourse and reasoning allows us to see this point more clearly and naturally leads to a nonmonotonic logic for if and oughts that elegantly removes the air of paradox from deontic conditionals.

The remainder of this section says a bit more about the puzzles surrounding deontic conditionals. §2 sketches a simple semantic analysis of conditionals and deontic ought that is in the spirit (but not the letter) of Kratzer’s seminal work on the meaning of modal expressions. The analysis is not unfamiliar—it is, plus or minus a bit, the one that is prominent in Kolodny’s and MacFarlane’s solution to the miners paradox—but it needs to be supplemented with an adequate notion of logical consequence for if and oughts. Thinking of validity in a classical fashion as necessary preservation of truth at a point of evaluation is good enough to block the miners paradox but does
not generalize to cover the remaining puzzles about deontic conditionals. A dynamic
approach to validity, in contrast, gets the facts about iffy oughts straight and this is
more than a sheer coincidence since it correctly predicts that the right logic for *ifs*
and *oughts* is nonmonotonic. (§3). §4 extends the dynamic proposal developed so far
to account for a few additional challenges. §5 offers a few general concluding remarks.

1.1 The Miners Paradox

Kolodny and MacFarlane consider the following scenario.\(^1\) Ten miners are trapped
either in shaft *A* or in shaft *B*, but we do not know which one. Water threatens to
flood the shafts. We only have enough sandbags to block one shaft but not both. If
one shaft is blocked, all of the water will go into the other shaft, killing every miner
inside. If we block neither shaft, both will be partially flooded, killing one miner.

<table>
<thead>
<tr>
<th>Action</th>
<th>if miners in <em>A</em></th>
<th>if miners in <em>B</em></th>
</tr>
</thead>
<tbody>
<tr>
<td>Block <em>A</em></td>
<td>All saved</td>
<td>All drowned</td>
</tr>
<tr>
<td>Block <em>B</em></td>
<td>All drowned</td>
<td>All saved</td>
</tr>
<tr>
<td>Block neither shaft</td>
<td>One drowned</td>
<td>One drowned</td>
</tr>
</tbody>
</table>

Lacking any information about the miners’ exact whereabouts, it seems right to say
that

(1) We ought to block neither shaft.

However, we also accept that

(2) If the miners are in shaft *A*, we ought to block shaft *A*,

(3) If the miners are in shaft *B*, we ought to block shaft *B*.

But we also know that

(4) Either the miners are in shaft *A* or they are in shaft *B*.

And (2)-(4) seem to entail

(5) Either we ought to block shaft *A* or we ought to block shaft *B*,

which contradicts (1).\(^2\) Thus we have a paradox.

Kolodny and MacFarlane argue that an adequate response to the puzzle must
reject the validity of modus ponens for deontic conditionals. Without it, the condi-
tional obligations articulated by (2) and (3) do not detach and thus we block the
classical derivation of the paradoxical conclusion (5) from (2)-(4). There is some rea-
son to think that rejecting modus ponens is a bit of an overreaction even if one agrees
with Kolodny’s and MacFarlane’s general strategy of blocking the argument on its
obvious logical form, but such reservations aside it is uncontroversial that at least
some semantic creativity will be required to make iffy oughts resistant to the miners
paradox.

\(^1\)See also the discussion from Parfit (1988, 2011), who credits Reagan (1980). A similar puzzle
involving “better” instead of “ought” is discussed by Dreier (2009).

\(^2\)Kolodny and MacFarlane evidently set aside the possibility of inconsistent deontic obligations.
Here and throughout it will be harmless to adopt this simplification since none of the paradoxes
discussed in this paper can be resolved by invoking the possibility of genuine moral dilemmas.
1.2 The Gentle Murder Paradox

Forrester observes that (6) and (7) are jointly consistent:

(6) Jones ought not murder Smith,

(7) If Jones murders Smith, he ought to murder Smith gently.

(6) articulates Jones’s primary obligation not to murder Smith. (7) says what Jones ought to do conditional on his violating his primary obligation. Intuitively, (6) and (7) are not only jointly consistent but also compatible with the possibility that Jones in fact murders Smith. But suppose that

(8) Jones murders Smith.

(7) and (8) entail

(9) Jones ought to murder Smith gently,

which hardly seems co-tenable with (6). Where did we go wrong?

Forrester thinks of the paradox as a problem for the Inheritance principle that ought is closed under logical entailment. Inheritance allows us to derive (10), which contradicts (6) and follows from (9) since even the gentlest of murders must count as a murder:

(10) Jones ought to murder Smith.

Not everyone thinks that Forrester has a conclusive case against Inheritance here: perhaps we can retain the principle and simply point to ambiguities in the logical structure of (9) to block to inference of (10). But regardless of what one wants to say about this issue, blocking the step from (9) to (10) does not resolve the conflict between (6) and (9). Forrester must admit that (6) forbids Jones to murder Smith gently or otherwise: even if one thinks that ought is not reliably closed under logical entailment, it should be uncontroversial that (6) entails

(11) Jones ought not murder Smith gently,

which now contradicts (9). The upshot: Forrester’s paradox is first and foremost a puzzle about the inferences we are inclined to draw from iffy oughts. Of course, one might once again insist that, all appearances notwithstanding, conditional obligations do not detach—crippling the inferential potential of iffy oughts is a reliable means for formally resolving paradoxes about conditional obligations—but this fails to make sense of the role that conditionals such as (7) play in discourse and reasoning. The point of contrary-to-duty obligations, after all, is to tell us what to do in case we neglect our duties so that we make the best of the bad situation to which our misdeed have led and it is, to say the least, hard to see how they can do so if such obligations fail to apply unconditionally to agents who do in fact neglect their duties. But this is just to say that our best semantics for ifs and oughts must allow us to derive unconditional from contrary-to-duty conditional obligations. So ideally, we can account for the joint consistency of (6) and (7) with (8) while allowing for contrary-to-duty obligations to detach.

3This alternative to Forrester’s diagnosis is explored by Castañeda (1985, 1986) and Sinnott-Armstrong (1985). Goble (1991) shows, conclusively I think, that their strategy does not generalize to cover variants of the gentle murder paradox, and endorses Forrester’s solution.
1.3 Chisholm’s Paradox

Forrester’s paradox is at its heart a puzzle about deontic conditionals that articulate contrary-to-duty obligations. Chisholm’s paradox crucially involves such conditionals as well. Consider the following:

(12) Jones ought to go to the aid of his neighbors,

(13) If Jones goes to the aid of his neighbors, then he ought to tell them he is coming,

(14) If Jones does not go to the aid of his neighbors, then he ought not tell them he is coming,

(15) Jones does not go to the aid of his neighbors.

It seems that (12)-(15) are consistent and that none of these statements logically implies any other one. Chisholm observes that von Wright’s (1951) classical deontic logic violates at least one these constraints regardless of how we formalize (12)-(15) but there is a more general reason for being interested in the case he describes. (12) and (13) are often taken to imply:

(16) Jones ought to tell his neighbors that he is coming.

But it also makes perfect sense to think that (13) and (15) imply

(17) Jones ought not tell his neighbors that he is coming,

which contradicts (16). So it seems that we need to deny at least one of the two types of inferences—often labeled “deontic detachment” and “factual detachment”, respectively—which is too bad since both are very plausible.4

1.4 Interlude

The literature on deontic paradoxes is, to say the least, extensive. In particular, Chisholm’s paradox is blessed with a large variety of proposed solutions that cannot be effectively dismantled here. My goal in this paper is to arrive at a framework that is general enough to take care of all our other paradoxes and lives happily with Kratzer’s basic insights into the semantics of modality. I will thus set aside proposals that are based on a wide scope analysis of deontic conditionals or follow von Wright (1956) in using a primitive binary conditional connective to represent conditional obligations.5 I will also set aside the possibility of challenging one or more premises of the arguments

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4See Loewer and Belzer (1983) and references therein for discussion. The labels “factual detachment” and “deontic detachment” go back to Greenspan (1975).

5Wide scope analyses treat the deontic modal as taking scope over a material conditional in a sentence like “If Jones goes to the aid of his neighbors, then he ought to tell them he is coming” and are thus hard to square with the insight that conditional antecedents are best understood as restrictors of modals. Notice also that wide-scoping does not help with Chisholm’s paradox; furthermore, it fails to be a promising strategy for resolving the miners paradox (see Kolodny and MacFarlane (2010, Sect. III.1)). Thomason (1981a) is an early critic of the use of primitive conditional connectives in representing conditional obligations.
under consideration since this strategy is not promising for the miners paradox and it is hard to deny that (6)-(8) and (12)-(15) are at least consistent.\footnote{The possibility of rejecting soundness while preserving validity in the case of the miners paradox is critically discussed by Kolodny and MacFarlane (2010, Sect. I.1) and also Charlow (forthcoming, Sect. 2). There is a broad consensus in the literature that (6)-(8) and (12)-(15) are consistent, but see Arregui (2010) for a dissenting voice.}

It is sometimes suggested that many deontic paradoxes can be resolved by paying close attention to the times of the obligations. The basic idea from Thomason (1981a,b) is that obligations depend on what possibilities are presumed to be open in context and since such possibilities change over time, so does what ought to be done.\footnote{See also, among others, Åqvist and Hoepelman (1981), van Eck (1982), Feldman (1986, 1990), and Loewer and Belzer (1983, 1986) for combinations of deontic and tense logic.}

The first bit of the suggestion—that what ought to be done varies with what counts as settled in deliberation—sounds right and will in fact be the starting point for the upcoming story about if's and oughts. Jones ought to go help his neighbors and also tell them that he is coming, and thus ought not tell them that he is not coming. But assume that Jones does not go: then in light of that information, it is no longer true that he ought to tell them that he is coming; rather, he should at least let them know that he is not coming. The use of deontic ought that is prominent here is what Thomason calls deliberative—the question is what ought to be done taking the facts as given—and it contrasts with non-deliberative uses in which the facts themselves may be subject to evaluation.\footnote{Schroeder (2011) argues that one must distinguish between a deliberative sense of ought that relates agents to actions and an evaluative sense that does not. The distinction he has in mind does not coincide with Thomason’s and so the question whether Schroeder is right, though important, need not detain us here.}

I shall work here under the assumption that Thomason’s distinction is real and that all the paradoxes we have considered above involve deliberative uses of deontic ought. A semantic solution to these paradoxes must then be sensitive to the flow of information in discourse and reasoning but it will not do, as Thomason suggests, to tie changes in what is settled to progression in time: possibilities in deliberation may shift without corresponding shifts in matters of fact, as in hypothetical reasoning. Taking a close look at the interaction between tense and modality is important but what matters for a solution to our paradoxes is the dynamics of contextual information rather than the dynamics of time.\footnote{The idea that deontic ought is sensitive to contextual information is also present in the discussion from Prakken and Sergot (1996, 1997), who talk about obligations as contextual obligations. They also present a timeless Chisholm scenario—their “fence example”—that puts pressure on views treating tense as the silver bullet against all paradoxes deontic.}

The proposed focus on deliberative interpretations of deontic ought raises the question to what extent the resulting semantics has any interesting implications for its non-deliberative uses. Thomason recognizes the appeal of a unified framework but does not suggest how it can be achieved. In a later section of this paper, I hope to show that general considerations about presupposition allow us to make some real progress towards a theory that explains the relation between deliberative and non-deliberative uses of deontic ought. The first task, however, is to put the basic semantic apparatus in place, which is the job of the next section.
2 Basics

The goal of this paper is to argue that an adequate model of the role of if s and oughts in discourse and reasoning must be based on a dynamic conception of logical consequence. But such a model must start with a reasonable semantics for conditionals and deontic modals. The view I will summarize here stays in already familiar territory. Kratzer treats modals as quantifiers over a contextually determined set of possible worlds and conditionals as modifiers of modals.\(^{10}\) Some modals are sensitive to the information that is taken for granted in discourse and reasoning. Epistemic modals like might and must are the most obvious examples, but we have also seen that it makes sense to think about deontic ought—at least when used deliberatively—as sensitive to contextual information as well. Information-sensitive modal operators are then just those whose domain depends on the information that is taken for granted in discourse and reasoning, and we can make this idea more precise by assigning to such modals truth-values at an index of evaluation with respect to some informational parameter, to be supplied by context and modelled in the usual fashion as a set of possible worlds. So following Kolodny and MacFarlane (2010), if \(w\) is a possible world and \(i\) is an informational parameter (modelled as a set of possible worlds), the semantics for such modals looks as follows:

\[
\begin{align*}
[\Box f \phi]^{w,i} &\text{ is true iff for all } w' \in f(i), [\phi]^{w',i} \text{ is true} \\
[\Diamond f \phi]^{w,i} &\text{ is true iff for some } w' \in f(i), [\phi]^{w',i} \text{ is true}
\end{align*}
\]

Here \(f\) is a selection function mapping information states to modal quantifier domains. Different modals require different selection functions. The modal quantifier domain of epistemic modals, for instance, is determined by an epistemic selection function \(e\), which for our purposes is just the identity function. Deontic ought is a universal quantifier ranging over a set of possible worlds that is provided by a deontic selection function \(d\).

The current proposal is not too far away from Kratzer’s semantic analysis of modality since we can think of informational parameters as modal bases and of selection functions as playing the role of ordering sources. How exactly contextual information conspires to determine modal quantifier domains is subject of an ongoing debate but this controversial issue need not detain us here.\(^{11}\) I will let \(d\) stand short for whatever turns out to be the best story about how context settles what is deontically ideal, and try to put this bit of abstraction from semantic details to good use by focussing on the question how a dynamic perspective on context-content interaction resolves our paradoxes about iffy oughts. I shall take it to be nonnegotiable, however, that our deontic selection function is realistic and—as long as this does not conflict with realism—only includes non-empty sets of possible worlds:

\(^{10}\)See Kratzer (2012) for an up-to-date discussion of her views about modals and conditionals and Kratzer (1977, 1981, 1991a,b) for some classical discussions.

\(^{11}\)For discussion see Cariani et al. (2011), Charlow (forthcoming), and Silk (2012), who all maintain that Kratzer’s classical proposal has trouble accounting for our intuitions about the miners scenario and explore various ways of amending it to do better. The need for such amendments is questioned by Dowell (forthcoming) and von Fintel (2012), as well as by Kratzer in unpublished notes. My story here is compatible with most things said in this debate, but it is worth repeating that I aim at an account that is attractive in part because it avoids appealing to multiple interpretations of ought in resolving our paradoxes.
Realism  For all $i$, $d(i) \subseteq i$

Non-Emptiness  For all $i \neq \emptyset$, $d(i) \neq \emptyset$

Realism requires that deontically ideal worlds are selected from the relevant informational parameter, which is just what we expect if we work with a deliberative interpretation of deontic ought. Non-emptiness effectively enforces the D-axiom of classical deontic logic, setting aside the possibility of appealing to inconsistent deontic obligations in explaining away the paradoxes under consideration.

Kratzer’s treatment of conditional antecedents as modifying the modal base in light of which we evaluate the consequent is captured as follows: in order to determine the truth value of a conditional we first strengthen the information in light of which we reason with the antecedent, and then evaluate the consequent in that light. The process of strengthening is defined as follows:

Strengthening  The result of strengthening $i$ with $\phi$, $i + \phi$, is defined as the intersection of $i$ and $[\phi]^i$, i.e. $i + \phi = i \cap \{w : [\phi]^w \text{ is true}\}$

An informational parameter $i$ affects the proposition expressed by $\phi$, and $\phi$ in turn strengthens $i$ by ruling out all possibilities that are incompatible with the proposition expressed by $\phi$ in $i$. The semantic analysis of conditionals is then:

$[\phi \Rightarrow \psi]^w_i$ is true iff $[\psi]^w_{i+\phi}$ is true

Here the assumption is that conditional consequents contain a modal operator. By default, the modal is a (perhaps implicit) epistemic necessity operator but in iffy oughts the modal is deontic.

The semantics presented here is flexible enough to account for our intuitions about the miners scenario but doing so requires some assumptions about the role that information strengthening plays for truth at a point of evaluation. Borrowing some useful terminology from Gillies (2010), the question is to what extent truth at a point is persistent:

Persistence

1. $\phi$ is $t$-persistent iff for all $w$, $i$ and $i' \subseteq i$: if $[\phi]^w_i$ is true then $[\phi]^w_{i'}$ is true

2. $\phi$ is $f$-persistent iff for all $w$, $i$ and $i' \subseteq i$: if $[\phi]^w_i$ is false then $[\phi]^w_{i'}$ is false

Sentences whose truth-values do not depend at all on what information is taken for granted in discourse and reasoning are, of course, both $t$-persistent and $f$-persistent. Epistemic might is $f$-persistent but not $t$-persistent: additional information may eliminate all $p$-worlds from the relevant informational parameter, thus switching the truth-value of $\Diamond_e p$ from true to false. Epistemic must is $t$-persistent but not $f$-persistent: additional information may eliminate all $\neg p$-worlds from the relevant informational parameter, thus switching the truth-value of $\Box_e p$ from false to true. And of course, this is all intimately tied to the fact that our epistemic selection function is monotonic: for all $i$, $i' \subseteq i$, $e(i') \subseteq e(i)$, which is not surprising since $e$ is just the identity function.
It is straightforward to verify that deontic \textit{ought} is neither t- nor f-persistent if our intuitions about the miners scenario are reliable. Remember that (1) and (2) from the miners paradox are jointly consistent (repeated):

(1) We ought to block neither shaft,

(2) If the miners are in shaft $A$, we ought to block shaft $A$.

To make sense of this, we need to say that, in light of the information we have in the miners scenario, “We ought to block neither shaft” is true and “We ought to block shaft $A$” is false. But strengthen the information from which we reason with the information that the miners are in shaft $A$: then in light of that information, “We ought to block neither shaft” is false and “We ought to block shaft $A$” is true, which just means that deontic \textit{ought} is neither t- nor f-persistent and that our deontic selection is neither monotonic nor anti-monotonic.

Denying t- and f-persistence of deontic \textit{ought} leaves room for the possibility that (2) and (3) are jointly compatible with (1) even if (4) is true. It is thus clear that not much creativity is needed to come up with a notion of logical consequence that predicts the miners paradox to be invalid on its obvious logical form. Classical logical consequence thinks of validity as necessary preservation of truth at a point of evaluation. The semantics outlined here defines truth at a point of evaluation—a possible world—with respect to some contextually provided informational parameter, and so the obvious thing to do is to follow Kolodny’s and MacFarlane’s path and think of logical consequence \textit{neoclassically} as necessary preservation of truth at some point of evaluation and with respect to some informational parameter.

**Neoclassical Logical Consequence** \[ \phi_1, \ldots, \phi_n \models_N \psi \text{ iff for all } w \text{ and } i \text{ such that } w \in i: \text{ if } w \in [\phi_1]^i \text{ and } \ldots \text{ and } w \in [\phi_n]^i, \text{ then } w \in [\psi]^i \]

The proviso that \( w \in i \) enforces that we only consider “proper” points of evaluation: this is to require that the context provides a modal base that is \textit{epistemically} in the sense that it does not carry any misinformation about the index of evaluation. Logical consequence thus defined blocks the inference of (5) from (2)-(4): given some \( w \) and \( i \) such that \( w \in i \) and \( w \in [inA \Rightarrow \Box_{d}blA]^i \), \( w \in [inB \Rightarrow \Box_{d}blB]^i \), and \( w \in [inA \lor inB]^i \), we know that both \( w \in [\Box_{d}blA]^{i+inA} \) and \( w \in [\Box_{d}blB]^{i+inB} \). But this, given the lack of f-persistence of deontic \textit{ought}, is compatible with both \( w \notin [\Box_{d}blA]^i \) and \( w \notin [\Box_{d}blB]^i \) (and even if \( i \subseteq [inA \lor inB]^i \)), which is just to say that the miners paradox is invalid on its obvious logical form.

A neoclassical supplement to our semantics for \textit{ifs} and \textit{oughts} successfully blocks the miners paradox on its obvious form, but it is important to notice that there are alternatives. Start with the feature of neoclassical consequence that Kolodny and MacFarlane hold to be a non-negotiable component of any adequate solution to the miners paradox: modus ponens is invalid for deontic conditionals. For suppose that \( w \in [\phi]^i \) and \( w \in [\phi \Rightarrow \psi]^i \) are true. It follows that \( w \in [\psi]^{i+\phi} \) is true, but that is compatible with \( w \notin [\psi]^i \) in case \( \psi \) fails to be f-persistent, which is just what we have if \( \psi \) is a deontically modalized sentence. Neoclassical consequence thus leaves something to be desired since it blocks factual detachment. But there is at least one
alternative notion of logical consequence that preserves the validity of modus ponens and nonetheless resolves the miners paradox.

Start with the observation that, given some contextually provided body of information, we can distinguish between what is true at a point of evaluation and what is accepted is the context—settled as true in light of the information that is contextually taken for granted.

**Acceptance** \( \phi \) is accepted in information state \( i \) iff for all \( w \in [\phi]_{w,i} \) is true.

Instead of thinking about validity neoclassically we might say that an inference is valid just in case whenever its premises are accepted in light of some informational parameter \( i \), so is its conclusion. An informational conception of logical consequence as proposed by Yalcin (2007) captures this idea:

**Informational Consequence** \( \phi_1, \ldots, \phi_n \models_I \psi \) iff for all \( i \): if \( \phi_1 \) is accepted in \( i \) and... and \( \phi_n \) is accepted in \( i \), then \( \psi \) is accepted in \( i \)

Yalcin shows that informational consequence is not without appeal and so it is reasonable to ask whether this notion of validity also has something useful to say about the miners paradox. To streamline the upcoming discussion, it is helpful to establish an alternative notion of logical consequence that preserves the validity of modus ponens and nonetheless resolves the miners paradox.

**Quasi-Validity** \( \phi_1, \ldots, \phi_n \models_Q \psi \) iff \( \Box_c \phi_1, \ldots, \Box_c \phi_n \models_N \psi \)

An argument is quasi-valid just in case its conclusion follows neoclassically whenever its premises are epistemically necessary.

**Fact 1** \( \phi_1, \ldots, \phi_n \models_I \psi \) iff \( \phi_1, \ldots, \phi_n \models_Q \psi \)

From left to right: suppose that \( \phi_1, \ldots, \phi_n \not\models_Q \psi \), then for some \( w \) and \( i \) such that \( w \in [\Box_c \phi_1]^i \) and... and \( w \in [\Box_c \phi_n]^i \) but \( w \not\in [\psi]^i \). So for all \( w' \in [\phi_i]^{w',i} \) is true and... and \( [\phi_n]^{w',i} \) is true. But since \( w \in i \), there is some \( w' \in i \) such that \( [\psi]^{w',i} \) is false. So there is some \( i \) such that \( \phi_1 \) is accepted in \( i \) and... and \( \phi_n \) is accepted in \( i \) but \( \psi \) is not accepted in \( i \), and thus \( \phi_1, \ldots, \phi_n \not\models_I \psi \).

From right to left: suppose that \( \phi_1, \ldots, \phi_n \not\models_I \psi \), then there is some \( i \) such that \( \phi_1 \) is accepted in \( i \) and... and \( \phi_n \) is accepted in \( i \) but \( \psi \) is not accepted in \( i \). So for all \( w, w \in [\Box_c \phi_1]^i \) and... and \( w \in [\Box_c \phi_n]^i \). Moreover, since \( \psi \) is not accepted in \( i \), it follows that for some \( w \in i \), \( w \not\in [\psi]^i \). So for some \( w \) and \( i \) such that \( w \in i : w \in [\Box_c \phi_1]^i \) and... and \( w \in [\Box_c \phi_n]^i \) but \( w \not\in [\psi]^i \), and thus \( \phi_1, \ldots, \phi_n \not\models_Q \psi \).

As Kolodny and MacFarlane observe, modus ponens is quasi-valid. Assume that \( w \in [\Box_c \phi]^i \) and \( w \in [\Box_c (\phi \Rightarrow \psi)]^i \). Then \( w \in [\phi \Rightarrow \psi]^i \) and thus \( w \in [\psi]^{i+\phi} \).

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12I follow here Yalcin in thinking of informational consequence and thus quasi-validity as a serious alternative to the neoclassical approach. In contrast, Kolodny and MacFarlane treat quasi-validity as a tool for explaining how certain inferences that are strictly speaking invalid may nonetheless be reasonable in certain situations. Their idea is also present in Stalnaker (1975) and I will come back to it at a later stage.
since $w \in \square \cdot [\[=\phi\]]^i$, $i + \phi = i$ and thus $w \in [\psi]^i$, as required. But the inference of (5) from (2)-(4) is not quasi-valid. To see this, notice that the truth-value of informational modals and conditionals entirely depends on the informational parameter. Such sentences are thus locally invariant in the following sense:

**Local Invariance**  \( \phi \) is locally invariant iff for all $i$: \( [\phi]^i = \emptyset \) or \( [\phi]^i = W \)

Whenever $\phi$ is locally invariant, $w \in [\phi]^i$ if and only if $w \in [\square \cdot \phi]^i$. In such cases strengthening $i$ with $\phi$ idles, i.e. $i + \phi = i$ and thus it is guaranteed that $w \in [\phi]^i + \phi$ as well. The simple observation is that the inference of (5) from (2)-(4) is quasi-valid just in case \( \square \cdot (inA \lor inB), inA \Rightarrow \square \cdot blA, inB \Rightarrow \square \cdot blA \lor \square \cdot blB \). But we already know that given the lack of f-persistence of deontic ought, the falsity of (5) is compatible with the truth of (2) and (3) even in case (4) is an epistemic necessity.

It follows from this and **Fact 1** that informational consequence avoids the miners paradox while preserving the validity of modus ponens. This is important because it shows that the first exhibit from our triad of puzzles about iffy ought does not present a compelling case against factual detachment: rejecting modus ponens for deontic conditionals is not necessary to block the derivation of our paradoxical conclusion. Factual detachment may or may not be a reliable rule of inference, but it takes more than the miners paradox to show that it is not.\(^{13}\) More importantly, however, is the general lesson that deciding on a semantics for conditionals and deontic modals does not settle the question which notion of logical consequence makes best sense of the role of iffy oughts in discourse and reasoning. Neoclassical consequence is not the only game in town: at a minimum, we know that informational consequence offers an alternative to defining validity as necessary preservation of truth at a proper index of evaluation.

At this stage, one may worry that there is not much to choose between neoclassical logical consequence and its informational alternative: both offer an escape route from the miners paradox, and the remaining differences are too murky to give rise to a substantial philosophical debate. I agree that there is not much to choose here but this is because none of the notions of logical consequence that we have considered so far offers a satisfying escape route from all our deontic paradoxes. For that, we need a model of deontic discourse and reasoning that pays proper attention to the dynamic mutual interaction between content and context, as I will demonstrate in the next section.

### 3 Toward Dynamic Consequence

Neoclassical as well as informational consequence offer an escape route from the miners paradox. This is not surprising since our semantics alone already explains how (1)-(4)

\(^{13}\)And it also takes more than the observation that modus tollens is unreliable for deontic conditionals, which Kolodny and MacFarlane take as evidence against modus ponens since it figures prominently in the classical derivation of modus tollens. Informational consequence does not support modus tollens: simply notice that "If the miners are in shaft $A$, we ought to block shaft $A$" and "It is not the case that we ought to block shaft $A$" are accepted at $i$ while "The miners are not in shaft $A$" is not. Dynamic logical consequence, as I have demonstrated elsewhere, also accounts for the invalidity of modus tollens while preserving the validity of modus ponens.
can be true while (5) is false. Things get more complicated once we look at Forrester’s paradox. The problem, remember, starts with the observation that (6) and (7) are jointly consistent:

(6) Jones ought not murder Smith  \[ \Box_d \neg m \]
(7) If Jones murders Smith, he ought to murder Smith gently  \[ m \Rightarrow \Box_d g \]

Furthermore, (6) and (7) are compatible with the assumption that (8) is true yet (7) and (8) together entail (9), which contradicts (6):

(8) Jones murders Smith  \[ m \]
(9) Jones ought to murder Smith gently  \[ \Box_d g \]

To avoid a paradox, the neoclassical approach must insist that (9) does not follow from (7) and (8), but this seems to conflict with the role that (7) plays in discourse, as Forrester himself observes. Suppose that I am advising Jones and point to rule (6). Jones responds that he is going to kill Jones anyway, and there is no more to be said on that matter. Pointing to rule (7), I tell him: “Well, you ought to kill him gently, then.” I have relied on modus ponens, and my reasoning as well as my morals would appear beyond reproach. Another worry about blocking the inference of (9) from (7) and (8)—one that has already been mentioned before but is worth repeating—is that it is entirely unclear how contrary-to-duty obligations can play their assigned role in everyday discourse and reasoning if they do not detach: to tell us what to do in case we neglect our duties so that we make the best of the bad situation to which our misdeed have led. In short, the neoclassical approach to logical consequence has nothing adequate to say in response to Forrester’s paradox.

Informational consequence avoids the charge of dismissing factual detachment for deontic conditionals: whenever a conditional and its antecedent are accepted in an information state, so is its conclusion, and this is exactly what it takes for modus ponens to be reliable on the informational conception of logical consequence. However, informational consequence does nothing to avoid the gentle murder paradox since it is committed to (6)-(8) being in fact inconsistent: (9) follows from (7) and (8) by modus ponens, but unless Jones is subject to inconsistent obligations we will not find an information state that settles both “Jones ought not murder Smith” and “Jones ought to murder Smith gently” as true. Unlike its neoclassical alternative, informational consequence thus preserves the intuitive inferential properties of deontic conditionals, but it runs into the other horn of the dilemma: the premises of Forrester’s paradox are predicted to be inconsistent, which is not the result we want.

Supplementing our semantics for *ifs* and *oughts* with a neoclassical or informational conception of logical consequence works fine for the miners paradox but leads to unacceptable results when it comes to the gentle murder paradox. This is interesting but the more important question is *why* the notions of validity that we have considered so far fail to deliver an adequate solution to Forrester’s puzzle. Start with the observation that both accounts can be interpreted as working under the assumption that the premises of an argument restrict the set of proper indices in light of which its conclusion is evaluated. Precisely, if Σ is the set of proper indices \( \langle w, i \rangle \) such that \( w \in i \), we can make the following observations about neoclassical and informational consequence:
Fact 2 \( \phi_1, \ldots, \phi_n \models_N \psi \text{ iff } \Sigma \cap [\phi_1] \cap \ldots \cap [\phi_n] \subseteq [\psi] \)

Fact 3 \( \phi_1, \ldots, \phi_n \models_I \psi \text{ iff } \Sigma \cap [\Box_c \phi_1] \cap \ldots \cap [\Box_c \phi_n] \subseteq [\psi] \)

Neoclassical and informational consequence thus have a purely eliminative conception of the role of premises in discourse and reasoning. This resonates well with the common conception of logical consequence as necessary preservation of truth at a point of evaluation but also brings in its wake a commitment to logical consequence being monotonic in the following sense:

Monotonicity If \( \phi_1, \ldots, \phi_n \models \psi \), then \( \phi_1, \ldots, \phi_n, \phi_{n+1} \models \psi \)

A commitment to monotonicity does not leave us much choice when it comes to reasoning about gentle murders: we must either deny that (9) follows from (7) and (8)—which is just what the neoclassicist does—or accept that (6)-(8) are inconsistent, which is just what we have if we adopt an informational approach to logical consequence. At a minimum, this observation shows that the neoclassical and informational problems with Forrester’s paradox are not coincidental but stem from a deep shared theoretical commitment. As a bonus, however, it also suggests that there is much to be gained from rejecting this commitment.\(^\text{14}\)

Challenging monotonicity promises to remove the air of paradox from Forrester’s puzzle since it allows us to say that even if (9) follows from (7) and (8) and contradicts (6), there is no contradiction since at no point in the argument are we committed to the conjunction of “Jones ought not murder Smith” and “Jones ought to murder Smith gently.” There is a point in the argument at which we hold that Jones ought not murder Smith, gently or otherwise. And under the supposition that Jones murders Smith, we hold that Jones ought to murder Smith gently. But under that supposition we no longer hold that Jones ought not murder Smith—additional information in discourse and reasoning may defeat prior deontic truths. To be precise, let \( \Gamma \) be the set of sentences consisting of (6) and (7), i.e. assume that \( \Gamma = \{ \Box_d \neg m, m \Rightarrow \Box_d g \} \). The hypothesis is that while \( \Box_d \neg m \) follows trivially from \( \Gamma \), it no longer follows from \( \Gamma \) if strengthened with the additional premise that Jones murders Smith: \( \Gamma \models \Box_d \neg m \) yet \( \Gamma', m \not\models \Box_d \neg m \). So even if \( \Gamma, m \models \Box_d g \) and \( \Box_d g \land \Box_d \neg m \models \bot \), (6)-(8) do not entail a contradiction since the right logic for if\( s \) and ought\( s \) fails to be monotonic.

A nonmonotonic perspective on if\( s \) and ought\( s \) promises an attractive escape route from Forrester’s paradox. This is important and deserves further elaboration but it is just as crucial to notice that we already have good reason to expect that a logic for if\( s \) and ought\( s \) fails to be monotonic. Monotonicity requires that additional information in discourse and reasoning preserves what has already been established, but we already know that deontic ought fails to be t-persistent and thus that information strengthening fails to preserve deontic truths. Persistence failures alone, of course,\(^\text{14}\)For previous discussions of the role of nonmonotonicity in deontic discourse and reasoning, see, e.g., Bonevac (1998), Hory (2003, 2007), as well as the papers collected in Nute (1997). As demonstrated momentarily, the story told here differs from previous nonmonotonic narratives in details, scope, and general aim. To cut a long story short: a nonmonotonic perspective on deontic ought is independently motivated and best elaborated by combining an intuitive semantics for deontic modals with a (no less intuitive) dynamic perspective on discourse and reasoning.
are not sufficient to arrive at a nonmonotonic conception of logical consequence: neoclassicism as well as its informational alternative define validity as preservation of truth at a possible world and in light a fixed informational parameter. Processing a premise in discourse and reasoning, then, does not strengthen the informational parameter in light of which subsequent claims are evaluated and thus persistence failures are barred from having any nonmonotonic effects in logical reasoning. Such static evaluation procedures for arguments stand in stark contrast with the familiar and very dynamic evaluation procedure for conditionals: to determine the truth-value of a conditional with respect to some informational parameter \( i \), one first strengthens \( i \) with the information carried by the antecedent and then evaluates the consequent in light of the output informational parameter \( i + \phi \). On the other hand, the semantics of conditionals also suggests a thoroughly dynamic perspective on discourse and reasoning. Let me explain.

The intuitive idea is that information does not remain static in discourse and reasoning. Premises in an argument do not only rule out certain possibilities but, just as importantly, strengthen the informational parameter in light of which subsequent claims are evaluated. This gives us a dynamic conception of logical consequence:

**Dynamic Logical Consequence** \( \phi_1, \ldots, \phi_n \models_D \psi \) iff for all \( w \) and \( i \) such that \( w \in i : \) if \( w \in [\phi_1]^i \) and \( \ldots \) and \( w \in [\phi_n]^{i + \ldots + \phi_{n-1}} \), then \( w \in [\psi]^{i + \ldots + \phi_{n-1} + \phi_n} \).

Dynamic logical consequence preserves the idea of logical consequence as necessary preservation of truth at a point of evaluation—a possible world—but in addition we keep track of the effects that premises have on the relevant informational parameter.

Dynamic validity entails informational validity since whenever an argument is dynamically valid, it will be so in the special case in which all its premises are accepted in the relevant informational operator and thus strengthening idles:

**Fact 4** If \( \phi_1, \ldots, \phi_n \models_D \psi \) then \( \phi_1, \ldots, \phi_n \models_I \psi \)

This is good news since it immediately shows that dynamic logical consequence is alike to the notions of logical consequence that we have considered so far in that it blocks the miners paradox on its obvious logical form. However, it also differs from its static competitors in several important ways. Unlike its neoclassical alternative, dynamic logical consequence preserves the validity of modus ponens:

**Fact 5** \( \phi \Rightarrow \psi, \phi \models_D \psi \)

Modus ponens is dynamically valid just in case for all \( w \) and \( i \) such that \( w \in i : \) if \( w \in [\phi \Rightarrow \psi]^i \) and \( w \in [\phi]^{i + (\phi \Rightarrow \psi)} \), then \( w \in [\psi]^{i + (\phi \Rightarrow \psi)} + \phi \). Since conditionals are locally invariant, this condition is equivalent to the requirement that whenever \( w \in [\phi \Rightarrow \psi]^i \) and \( w \in [\phi]^i \), then \( w \in [\psi]^{i + \phi} \). But our semantics for conditionals guarantees that whenever \( w \in [\phi]^i \), then \( w \in [\psi]^{i + \phi} \), which is just to say that modus ponens is dynamically valid. And this is not a purely formal result but captures

\(^{15}\text{See also Gillies (2009); for some for some alternative dynamic conceptions of logical consequence, see Veltman (1985, 1996).}\)
something important about the connection between conditionals and information in discourse and reasoning: if a conditional is true in light of some body of information \(i\), its consequent is true in light of the result of strengthening \(i\) with its antecedent. But of course, processing a premise \(\phi\) in discourse and reasoning crucially involves strengthening the contextual information with \(\phi\). The validity of modus ponens, then, is a natural outcome of the concord between the evaluation procedure for conditionals and the way information is processed in discourse and reasoning. It is only when we postulate a mismatch between these two—as the neoclassicists do—that modus ponens becomes something mysterious.

We have seen that dynamic validity is alike to its informational alternative in that it licenses factual detachment and thus accounts for the intuition that (9) follows from (7) and (8). At the same time, it avoids the problems of informational consequence by preserving the intuition that (6)-(8) are consistent. The first crucial observation is that dynamic logical consequence avoids a purely eliminative conception of the role of premises in discourse and reasoning and thus a commitment to monotonicity. To get this into clearer view, it is helpful to define an update function \([\cdot]\) on sets of proper world-information state pairs, as follows:

**Update** Consider any \(\phi\) and \(\sigma \subseteq \Sigma\). An update on a set of proper indices is a function \([\cdot]: \mathcal{P}(\Sigma) \rightarrow \mathcal{P}(\Sigma)\) is defined as follows:

\[
\sigma[\phi] = \{\langle w, i + \phi \rangle : \langle w, i \rangle \in \sigma \land w \in [\phi]^i\}
\]

Updating a set of proper indices \(\sigma\) with \(\phi\) proceeds by first eliminating all elements of \(\sigma\) in light of which \(\phi\) is false, and then strengthening the informational parameters of the remaining elements with \(\phi\). Notice that \([\cdot]\) is well-defined since whenever \(w \in i\) and \(w \in [\phi]^i\), then \(w \in i + \phi\).

Dynamic logical consequence can then be captured as follows:

**Fact 6** \(\phi_1, \ldots, \phi_n \vdash_D \psi\) iff \(\Sigma[\phi_1] \ldots [\phi_n] \subseteq [\psi]\)

The crucial observation is that updating a set of proper indices is not guaranteed to be a purely eliminative affair since it is not guaranteed that whenever \(\langle w, i \rangle\) is an element of \(\sigma\), so is \(\langle w, i + \phi \rangle\). Dynamic logical consequence, then, is not wedded to a purely eliminative conception of premises in discourse and reasoning. And noneliminative effects are a real possibility whenever some formulas of our language fail to be t-persistent: whenever \(\phi\) fails to be t-persistent, we can expect that there are \(w, i, i' \subseteq i\) such that \(\langle w, i \rangle \in \Sigma[\phi]\) but \(\langle w, i' \rangle \notin \Sigma[\phi]\), and thus updating \(\Sigma[\phi]\) with another formula \(\psi\) may very well re-introduce indices eliminated by the update with \(\phi\). For instance, let \(i = W\) and consider \(w \in [\bar{p}]^i\): then \(\langle w, i + \bar{p} \rangle \notin \Sigma[\diamond_e \neg p]\). But since \(\langle w, i \rangle \in \Sigma[\diamond_e \neg p]\) and \(w \in [\bar{p}]^{i+\diamond_e \neg p}\), \(\langle w, i + \bar{p} \rangle \in \Sigma[\diamond_e \neg p][\bar{p}]\) and so updating \(\Sigma[\diamond_e \neg p]\) with \(\bar{p}\) has a noneliminative effect.

The observed noneliminative effects also illustrate that dynamic logical consequence fails to be monotonic, as desired. It follows from the local invariance of *might* that \(\diamond_e \neg p \vdash_D \diamond_e \neg p\), but our earlier observation that \(\langle w, i + \bar{p} \rangle \in \Sigma[\diamond_e \neg p][\bar{p}]\) shows that \(\diamond_e \neg p, \bar{p} \not\vdash_D \diamond_e \neg p\), which just delivers the following result:
Fact 7 Dynamic logical consequence is nonmonotonic.

And this, bear in mind, not by fiat but by the interaction between a dynamic evaluation procedure for arguments with independently motivated persistence failures for informational modals.

All of this allows us to endorse factual detachment without regrets. Specifically, a dynamic supplement to our semantics for *ifs* and *oughts* resolves the stubborn paradox of gentle murder. Start again with (7) and (8), i.e. assume that \( \Gamma = \{ \Box_d \neg m, m \Rightarrow \Box_d g \} \). Then trivially \( \Gamma \models_D \Box_d \neg m \) and \( \Gamma, m \models_D \Box_d g \) by modus ponens. Also, \( \Box_d \neg m \land \Box_d g \vdash_D \bot \) since gentle murders are murders and given the non-emptiness constraint on our deontic selection function. Nonetheless, there is no paradox—(6)-(8) are dynamically consistent—for even though “Jones ought not murder Smith” is true in light of our initial state of information, it is no longer true in light of the result of strengthening that state with the information that Jones murders Smith: if \( w \in [\Box_d \neg m]^i, w \in [m \Rightarrow \Box_d g]^i, \) and \( w \in [m]^i, \) then \( w \in [\Box_d g]^{i+m} \) but \( w \notin [\Box_d \neg m]^{i+m}, \) which is just to say that \( \Gamma, m \not\models_D \Box_d \neg m \) and thus \( \Gamma, m \not\models_D \bot. \)

Dynamic logical consequence preserves factual detachment for contrary-to-duty obligations without negative side effects since it pays proper attention to how information in discourse and reasoning affects contextual parameters that are relevant for evaluating *ifs* and *oughts*. Information strengthening may license factual detachment but at the same time defeat prior commitments whenever we have lack of t-persistence. These two features conspire in the dynamic solution to the gentle murder paradox: the assumption that Jones murders Smith licenses detachment of a contrary-duty-obligation while at the same time preserving consistency through defeating a conflicting primary obligation.

The upshot of the discussion so far is that a dynamic supplement to our semantics for *ifs* and *oughts* has a lot going for it. It matches the evaluation procedure for arguments against what is commonly recognized to be our best semantic take on the evaluation procedure for conditionals. It licenses factual detachment for deontic conditionals and thus preserves the intuitive role of conditional obligations in discourse and reasoning. And it does so while avoiding the notorious gentle murder paradox by taking a nonmonotonic perspective on deontic discourse and reasoning. This, I submit, is sufficient reason to see how a dynamic story for *ifs* and *oughts* can be further elaborated to take a few more crucial data into account. This is what I will do in the next section.

4 Bells and Whistles

4.1 Chisholm’s Paradox

Chisholm’s paradox starts with the observation that (12)-(15) are consistent and none of these statements logically implies any other one. This is especially puzzling since (12) and (13) seem to imply (16) while (14) and (15) seem to imply (17) (repeated):

(12) Jones ought to go to the aid of his neighbors,

(13) If Jones goes to the aid of his neighbors, then he ought to tell them he is coming,
If Jones does not go to the aid of his neighbors, then he ought not tell them he is coming,

Jones does not go to the aid of his neighbors,

Jones ought to tell his neighbors that he is coming,

Jones ought not tell his neighbors that he is coming.

We already saw that dynamic logical consequence preserves modus ponens and thus predicts the inference of (17) from (14) and (15). The inference of (16) from (12) and (13)—deontic detachment—is not licensed so far but easy to enforce by imposing the following constraint on our deontic selection function:

**Weak Stability**  For all \(i, i' \subseteq i\), if \(d(i) \subseteq i'\), then \(d(i) \subseteq d(i')^{16}\)

Weak Stability requires that strengthening with what is considered to be deontically ideal preserves deontically ideal worlds: doing what one ought to do at best absolves us from some our duties but does not create any new ones. This requirement lives happily with what I said before in response to the miners and the gentle murder paradox, and it is just what we need to license the inference of (16) from (12) and (13):

**Fact 8**  Given Weak Stability, \(\Box_d \text{go} \Rightarrow \Box_d \text{tell} \models D \boxminus \Box_d \text{tell}\)

This holds just in case for all \(w\) and \(i\) such that \(w \in i\): if \(w \in [\Box_d \text{go}]^i\) and \(w \in [\text{go} \Rightarrow \Box_d \text{tell}]^{i+\text{go}}\), then \(w \in [\Box_d \text{tell}]^{(i+\text{go})+\{\text{go} \Rightarrow \Box_d \text{tell}\}}\). Since conditionals and deontic ought are locally invariant, this condition reduces to the requirement that whenever \(w \in [\Box_d \text{go}]^i\) and \(w \in [\text{go} \Rightarrow \Box_d \text{tell}]^i\), then \(w \in [\Box_d \text{tell}]^{i+\text{go}}\). Observe that \(d(i) \subseteq i\) because of Realism and suppose that \(w \in [\Box_d \text{go}]^i\) and \(w \in [\text{go} \Rightarrow \Box_d \text{tell}]^i\); then \(d(i) \subseteq [\text{go}]^i\) and thus \(d(i) \subseteq i + \text{go}\). Weak Stability then guarantees that \(d(i) \subseteq d(i + \text{go})\). But \(d(i + \text{go}) \subseteq [\text{tell}]^{i+\text{go}}\) and so \(d(i) \subseteq [\text{tell}]^{i+\text{go}}\). Since \(\text{tell}\) is \(f\)-persistent, \(d(i) \subseteq [\text{tell}]^i\) and thus \(w \in [\Box_d \text{tell}]^i\), as required.

The fact that we can find constraints on our deontic selection that license deontic detachment is perhaps not terribly surprising, and in any case those who are

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16Kratzer’s semantics for deontic modals is committed to the following constraint on the deontic selection function, which Charlow (forthcoming) aptly labels “Stability.”

**Stability**  For all \(i, i' \subseteq i\), if \(w \in d(i)\) and \(w \in i'\), then \(w \in d(i')\)

In words, a world that is deontically ideal—minimal in light of some ordering source—remains deontically ideal under strengthening as long as the world is not ruled out by the additional bit of information. It is a well-worn story that Stability is problematic when it comes to accounting for our intuitions about the miners paradox. Whatever the merit of this concern, notice that the requirement introduced to take care of Chisholm’s paradox is strictly weaker than the stability criterion that is integral to Kratzer’s semantics. Kratzer’s criterion requires that a deontically ideal world is guaranteed to be preserved under any form of strengthening as long as it is not ruled by the additional bit of information. In contrast, Weak Stability merely requires that a deontically ideal world is guaranteed to be preserved under a specific form of strengthening, i.e. one that does not exclude any deontically ideal world whatsoever.
committed to a neoclassical or informational approach to logical consequence can do the same. What is worth mentioning, however, is that we can endorse both factual and deontic detachment while avoiding Chisholm’s paradox. And once again it is the already familiar nonmonotonicity of dynamic logical consequence that does the trick.

Deontic detachment licenses the inference of (16) from (12) and (13), but this inference is defeated by the premise (15) that triggers detachment of the contrary-to-duty obligation articulated by (17). So to be precise, assume that \(\Sigma_t = \{\Box_d \neg go, \neg go \Rightarrow \Box_d \neg tell\}\) and that \(\Sigma' = \{\neg go, \neg go \Rightarrow \Box_d \neg tell\}\): then \(\Sigma \models_D \Box_d tell\) and \(\Sigma' \not\models_D \Box_d \neg tell\) as required by deontic and factual detachment, respectively. Yet there is no contradiction, for even though \(\Sigma \models_D \Box_d tell\), we also have \(\Sigma, \neg go \not\models_D \Box_d tell\), which is compatible with deontic detachment since our deontic selection function is required to be realistic and thus \(\Sigma, \neg go \not\models_D \Box_d go\). So under the assumption that Jones does not go, he ought not tell his neighbors that he is coming, but that is consistent with the intuition that he ought to tell his neighbors that he is coming in case he ought to go and also ought to tell them if he goes. What makes all this possible is that additional information in discourse and reasoning may defeat prior deontic truths and thus, in turn, inferences such as those licensed by deontic detachment.

4.2 Order-Sensitivity and Realism

This section says a bit more about two salient features of the semantic framework suggested so far: the sensitivity of logical consequence to the order of premises and the assumption of a realistic deontic selection function. Dynamic logical consequence, as I have shown, offers an attractive perspective on deontic discourse and reasoning since it allows us to preserve intuitive rules of inference for deontic conditionals as well as the consistency of the premises in Forrester’s and Chisholm’s paradox. But, as so often in dynamic approaches to discourse and reasoning, order matters. It is easy to verify that reversing the order of premises in Forrester’s paradox results in a contradiction, i.e. \(m, m \not\models \Box_d go, \Box_d m \models_D \bot\) (and similarly for Chisholm’s paradox). The question whether this is a damaging result is quite subtle and must be left to another day, but at a minimum it is fair to worry that we have identified an unwelcome limitation on the explanatory power of the dynamic approach outlined so far. The good news is that one can arrive at a slightly modified conception of logical consequence that preserves the key ideas of my dynamic proposal and sidesteps the issue of order-sensitivity. Let me explain.

So far I have not said how conjunction works in our semantics for deontic ought. There are several options but here it actually pays off to stay traditional and define conjunction in the following way:

\[
[\phi \land \psi]^{w,i} \text{ is true iff } [\phi]^{w,i} \text{ is true and } [\psi]^{w,i} \text{ is true}
\]

Conjunction thus defined is “internally static” in that both conjuncts are evaluated with respect to the same informational parameter.\(^{17}\) It is therefore commutative and

\(^{17}\)For comparison, consider the following alternative treatment of conjunction:

\[
[\phi \land \psi]^{w,i} \text{ is true iff } [\phi]^{w,i} \text{ is true and } [\psi]^{w,i+\phi} \text{ is true}
\]

Here the second conjunct is evaluated with respect to the result of strengthening the relevant informational parameter with the first conjunct. This “internally dynamic” conception of conjunction

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so a sentence like (18) is consistent regardless of the order of conjuncts:

(18) Jones murders Smith and he ought not murder Smith.

It is straightforward to find a possible world $w$ at which Jones murders Smith and an informational parameter $i$ in light of which Jones ought not murder Smith, and this is all that we need to find a point of evaluation at which (18) is true, regardless of the order of conjuncts.

The reason behind this exercise about conjunction is that it offers a slightly modified dynamic perspective on logical consequence. Start with adopting the familiar dynamic story for single-premise arguments: $\phi$ entails $\psi$ just in case for all $w$ and $i$ such that $w \in [\phi]^i$, then $w \in [\psi]^{i+\phi}$. Then say that a multi-premise argument is valid just in case the conclusion follows from the conjunction of its premises.

Precisely:

Dynamic Logical Consequence, v.2 Define validity for single-premise arguments and then generalize to cover multi-premise arguments:

1. $\phi \models_D^w \psi$ iff for all $w$ and $i$ such that $w \in [\phi]^i$, then $w \in [\psi]^{i+\phi}$
2. $\phi_1, \ldots, \phi_n \models_D^w \psi$ iff $\phi_1 \land \ldots \land \phi_n \models_D^w \psi$

This version of dynamic logical consequence shares the virtues of its predecessor. Above all, it resolves Forrester’s and Chisholm’s paradox while preserving factual as well as deontic detachment, and for familiar reasons. Factual information continues to play its usual dynamic role of strengthening the informational parameter in light of which the conclusion of an argument is evaluated. In combination with the independently plausible failure of $t$-persistence for deontic $ought$, this yields the desired nonmonotonic effects. But in addition, the setup for conjunction guarantees that the premises of Forrester’s and Chisholm’s paradox are dynamically consistent regardless of the order of premises.

I am about to conclude that a dynamic perspective on discourse and reasoning offers an attractive escape route from a number of notorious paradoxes about deontic conditionals. Before that, let me make a few brief remarks about the choice of a realistic selection function in modelling the semantics of deontic $ought$. This, remember, yields a deliberative interpretation of deontic $ought$ that articulates what ought to be done taking the facts as given and thus differs from a non-deliberative interpretation on which the facts themselves may be subject to evaluation. It should be uncontroversial that a deliberative interpretation is prominent across our deontic paradoxes: it makes good sense of the miners paradox, and it is the only one that leads to potential commutativity failures. The notion of an internally dynamic (or static) connective goes back at least to Groenendijk and Stokhof (1991), whose discussion is concerned with the issue of variable binding.

Another way of stating this revised version of dynamic logical consequence is the following:

Dynamic Logical Consequence, v.2 (Rewrite) $\phi_1, \ldots, \phi_n \models_D^w \psi$ iff for all $w$ and $i$ such that $w \in [\phi_1]^i$ and... and $w \in [\phi_n]^i$, then $w \in [\psi]^{i+\phi_1} \land \ldots \land (i+\phi_n)$

There is, as far as I can see, not much to choose here, though I find the version stated in the main text more intuitive.
leaves room for conditionals articulating contrary-to-duty obligations to be true (non-deliberatively speaking, Jones ought not murder Smith gently regardless of whether he murders Smith). All of this puts our choice of a realistic deontic selection function on stable conceptual grounds but we still need a story that explains how deliberative and non-deliberative interpretations of deontic ought are generated in discourse and reasoning. Let me briefly outline how such a story might go.

I submit that a deliberative interpretation is the default, and this is so for obvious pragmatic reasons: as Stalnaker (1975) observes, a speaker is normally concerned only with possible worlds that are compatible with what is taken for granted, and so it is the normal expectation that the deontic selection function is realistic. This default may be overridden in various ways but one case deserves special attention. Intuitively, a deliberative use of ought is only felicitous in case the prejacent is something to deliberate over, i.e. only if the prejacent is not settled in one way or the other. So what I suggest is that deliberative uses of deontic ought come with a special entertainability presupposition: both the prejacent as well as its negation must be an epistemic possibility in light of the relevant informational parameter. If this condition is not fulfilled, the deontically modalized judgment may only receive a non-deliberative interpretation, which is just to say that the deontic selection function may reach outside the informational parameter.

Thomason (1981a) observes that non-deliberative uses of deontic ought involve a process of “bracketing off” certain facts. He captures this in a tense framework but here we can we speak more generally of non-deliberative interpretations as involving a bit of counterfactual reasoning: interpreting deontic ought non-deliberatively involves a process of temporally suspending belief that the prejacent is settled in one way or the other. To make this more precise, we may follow Grove (1988) in thinking of an information state $i$ as surrounded by a system of spheres ordered by the subset relation, where each sphere represents a degree of closeness or similarity to $i$. Such a system of spheres determines what counts as minimal revision to accommodate a certain bit of information. To evaluate a use of deontic ought non-deliberatively with respect to $i$ is then to ask whether the judgment is true with respect to another informational parameter $i'$, where $i'$ is the smallest sphere around $i$ at which the prejacent is not settled in one way or the other. We thus arrive at a uniform semantics for deliberative and non-deliberative interpretations of deontic ought: both are universal quantifiers over a domain that is determined by an informational parameter, the difference being what information is taken as deontically relevant.

The (anything but trivial) exercise of extending a dynamic framework for modals to capture the facts about presupposition and accommodation has been executed elsewhere and need not be repeated here. Instead, let me point out that the outlined suggestion is not only intuitive but has considerable explanatory power. Above all, it avoids the unwelcome result that (19) is trivially true:

(19) If Jones murders Smith, he ought to murder Smith.

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19 Entertainability presuppositions also play a major role in von Fintel’s (2001) and Gillies’s (2007) treatment of counterfactuals as strict conditionals over a dynamically developing domain of quantification.

20 See Beaver (2001) for seminal work, who also offers a careful analysis of previous discussions in the literature.
Together with our semantics for conditionals, a realistic deontic selection function enforces the truth of (19). But given what we said about entertainability presuppositions, a deliberative interpretation of the consequent is not available. Instead, the consequent must be evaluated with respect to an informational parameter that does not settle whether Jones murders Smith, and of course we then expect that the consequent is false. In fact, we predict, correctly I think, that (19) is slightly weird since evaluating the consequent requires that we ignore the information carried by the antecedent.\footnote{Frank (1997) and Zvolenszky (2002) argue that conditionals like the one in (19) are problematic for Kratzer’s treatment of modals and conditionals. In response, it is sometimes suggested that such conditionals are systematically ambiguous: on one reading, the if-clause is a restrictor of the overt deontic modal; on another reading, the if-clause restricts the domain of a covert quantifier that defaults to epistemic necessity. See Geurts (2004) for seminal discussion and, more recently, Kratzer (2012, Sect. 4.6). While there is much to like about this strategy, I choose a different path here that is not limited to avoiding a specific problem about deontic conditionals.}

Another notable result of the current setup is that it explains why a commitment to (9) does not entail a commitment to (10) in Forrester’s paradox (repeated):

(9) Jones ought to murder Smith gently,

(10) Jones ought to murder Smith.

The familiar observation is that (9) strikes us as true under the assumption that Jones murders Smith, but in that context the entertainability presupposition carried by (10) is violated. Of course, we may once again adopt a non-deliberative interpretation, but this requires that (10) is evaluated with respect to an informational parameter that leaves it open whether Jones murders Smith. In that context (10) as well as (9) are false.

The upshot of this section is that we can adopt a dynamic story about if and ought without potentially harmful side-effects. It is unproblematic to avoid the result that consistency is order-sensitive and we can leave room for non-deliberative interpretations of deontic ought while preserving a uniform semantics for deontic modals. All of this should give us even more confidence that dynamic approach developed here is on the right track.

5 Concluding Remarks

The goal of this paper has been to look for a semantic approach to conditionals and deontic modals that does not only avoid the miners paradox but also Forrester’s and Chisholm paradox while preserving intuitive rules of inference for deontic conditionals. A treatment of deontic ought as an informational modal is on the right track but requires that we re-think the notion of logical consequence. Neoclassical and informational consequence resolve the miners paradox but fail to deliver adequate responses to the more classical paradoxes from Forrester and Chisholm, and this is not a coincidence since both notions of validity are committed to a monotonic conception of logical consequence. A dynamic conception of logical consequence, in contrast, naturally leads to a nonmonotonic logic for deontic discourse and reasoning and this, I have
argued, offers an attractive escape route from the puzzles surrounding conditionals articulating contrary-to-duty obligations.

In arguing for a dynamic approach to deontic discourse and reasoning, I have put a lot of weight on the observation that dynamic logical consequence avoids our deontic paradoxes while preserving factual and deontic detachment for iffy oughts. It strikes me as uncontroversial that this is the best result one might hope for, and neither neoclassical nor informational consequence have something similar to offer. But one might still worry that I have overstated my case. The complaint about neoclassical consequence was that it fails to deliver the validity of factual detachment and thus does not account for the role that conditionals articulating contrary-to-duty obligations play in everyday discourse and reasoning. However, and as I mentioned earlier, modus ponens is quasi-valid, which is just to say that it is neoclassically valid whenever the antecedent is epistemically necessary: $\phi \Rightarrow \psi$, $\square_{\omega} \phi \models_{N} \phi$. It follows that contrary-to-duty obligations detach on the neoclassical picture whenever it is settled that the relevant primary obligation is violated, and one might insist that this is all we need to account for the role that such conditional obligations play in discourse and reasoning.\footnote{This response is very much in the spirit of Kolodny’s and MacFarlane’s view on how life goes on without modus ponens. The result that modus ponens is generally invalid but holds whenever the antecedent is settled can also be found in the frameworks developed by Belnap et al. (2001), Hory (2001) and Loewer and Belzer (1983, 1986), who interpret settledness in a temporal framework.}

We are well advised not to be satisfied with what the neoclassicist has to offer. Above all, it is nonnegotiable that plain factual information triggers detachment, and this still remains a mystery on the neoclassical picture. To get the problem into clearer view, consider again the miners scenario. Alex and Mary are discussing what to do; Mary is a bit better informed about the miners’ whereabouts than Alex:

\begin{align*}
(20) & \text{Alex: We ought to block neither shaft.} \\
& \text{Mary: But the miners are in shaft A.} \\
& \text{Alex: Oh, so we ought to block shaft A, then.}
\end{align*}

Here the factual information that the miners are in shaft A triggers detachment, and it is just as easy to imagine discourses in which plain factual information allows us to infer contrary-to-duty obligations. Restricting modus ponens to cases in which the antecedent is epistemically necessary, then, does not give us everything we need to make sense of deontic conditionals.

Of course, there is some room for pragmatic maneuvers here. The obvious response on behalf of the neoclassicist takes some inspiration from Stalnaker’s (1978) seminal work on assertion and maintains that Mary’s utterance modifies the conversational context: it strengthens the common ground between Alex and Mary by eliminating all possible worlds at which the miners are in shaft A. With respect to the modified context, it is epistemically necessary that the miners are in shaft A, and so once again we have a case in which even the neoclassicist can predict detachment.

The outlined pragmatic rejoinder is not unreasonable but what does the trick is the assumption that epistemic must interacts dynamically with factual information in discourse: the assertion that the miners are in shaft A updates the common ground, triggering a transition from a state of information according to which the miners might be in shaft A but also might be in shaft B (and thus we ought to block neither...
shaft) to one according to which the miners must be in shaft A (and thus we ought to block shaft A). The problem for the neoclassicist is that there is no reason at all to think that the dynamics of must is a merely pragmatic affair—it is just as robust in reasoning, as the following example demonstrates:

(21) a. The miners are in shaft A or they are in shaft B.
   b. They are not in shaft B.
   c. So, they must be in shaft A.

This is as an intuitive entailment as we are likely to find even if (21a) and (21b) occur in a merely hypothetical context, and to make sense of it one needs to admit that the information that the miners are not in shaft B eliminates the epistemic possibility that they are in shaft B. In short, \( \phi \) defeats \( \Box_c \neg \phi \) not only in everyday discourse but also in everyday reasoning. What started out as a pragmatic response to the shortcomings of neoclassicism, then, collapses into an argument for the general validity of factual detachment: \( \phi \) entails \( \Box_c \phi \) and since \( \Box_c \phi \) together with \( \Box \phi \Rightarrow \Box d \psi \) entail \( \Box d \psi \), we get factual detachment. There is, however, a bit more to learn from this exercise.

The preceding considerations matter since we are now in a position to diagnose the troubles with neoclassicism. There is nothing wrong with the neoclassical contention that \( \Box \phi \Rightarrow \psi \) together with \( \Box_c \phi \) entail \( \psi \), but everything wrong with its contention that \( \Box \phi \Rightarrow \psi \) together with \( \phi \) do not entail \( \psi \). These facts are related because a commitment to \( \phi \) brings in its wake a commitment to \( \Box_c \phi \), and not only in discourse but also in reasoning. This is what we have on the dynamic conception of logical consequence, i.e. \( \phi \vdash_D \Box_c \phi \), but not on the neoclassical view since \( \phi \not\vdash_N \Box_c \phi \). Neoclassicists are thus right to say that modus ponens is valid whenever the antecedent is epistemically necessary but at the same time overlook the interaction between factual information and epistemic necessity: the neoclassical failure to get the basic facts about deontic ought straight (factual detachment) is grounded in its failure to capture the dynamics of epistemic might and must. Once again, we arrive at the dynamic perspective on discourse and reasoning as the right supplement to information-sensitive semantics.

Combining insights from dynamic semantics with recent work on deontic modality allows us to make some real progress towards a satisfying solution for a number of notorious paradoxes about deontic conditionals. Without doubt, there are many more issues about deontic modality to explore. Above all, I have abstracted from the question how exactly contextual information determines what is deontically ideal and set aside interactions between tense and deontic modality. I have also not addressed the possibility of genuine moral dilemmas, the topic of weak vs strong deontic necessity, or issues pertaining to degree of value, comparing one option to another, and the number of deontic notions connected to these ideas.\(^{23}\) But I submit that as soon as things get iffy in the realm of deontic modality, a dynamic perspective is the right one to take.

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\(^{23}\)For a discussion of strong vs weak necessity in deontic modality, see von Fintel and Gillies (2008). For a discussion of supererogation and related notions, see McNamara (1996a,b).
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